Chapter V

A Review of Methods in the Control of Continuous Systems

5.1 INTRODUCTION

As pointed out in Chapter 2, control of continuous systems represents a specific situation which requires a specific approach. Mainly, if the differential operator of the control problem has become nonselfadjoint, one is faced with the following difficulty: the possible occurrence of spillover. Spillover is the incidence of instability by virtue of higher, unstable modes, which have not been considered in the design of the control.

In view of the danger of spillover, a modal approach is not advisable, and new methods may have to be developed. How that can be done, will be shown to a certain extent in this chapter. Essentially, three possibilities will be discussed: In the first case, the operators characterizing the problem will be considered directly, and one will try to come to conclusions on the behaviour of the control system, taking the properties of the operators into account. In the second case, a closed form solution to the problem will be sought, thus enabling one to avoid the modal approach. In the third case, a liapunov like approach will be presented for continuous systems with nonselfadjoint operators.
5.2 CONSIDERATIONS OF EXISTENCE AND PERFORMANCE OF A CONTROL

Consider the control of a plate subjected to loading transversal to the plate's surface. Let first the motion of the uncontrolled plate be investigated. Assuming that the plate is a thin one, the equation of motion for this plate reads

$$\rho h \ddot{w} + N \nabla^4 w = q(x,y,t) .$$  \hspace{1cm} (5.1)

In (5.1), \( \rho \) is the density per unit volume, \( h \) the plate's constant thickness, \( N \) the flexural rigidity, \( w(x,y,t) \) the transversal deflection of the plate's middle surface, \( q \) the distributed load, \( x \) and \( y \) the spatial coordinates, and \( t \) the time. Moreover,

$$\dot{w} = \frac{\partial^2 w}{\partial t^2} , \quad \nabla^4 w = \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} .$$  \hspace{1cm} (5.2)

In addition to (5.1), boundary and initial conditions must be specified as to define the problem of the plate's motion definitely. In order to simplify the situation, let the initial conditions be homogeneous ones. The boundary conditions may be indicated by the expression

$$U[w]_B = 0 ,$$  \hspace{1cm} (5.3)

where \( U \) is a "vector" of operators on \( w \) with respect to \( x \) and \( y \) (not \( t \)), and where the resulting expression \( U[w] \) is to be taken at the boundary \( B \) (contour) of the plate.

It is known that the operator in (5.1), i.e.,

$$D = \rho h \frac{\partial^2}{\partial t^2} + N \nabla^4 ,$$  \hspace{1cm} (5.4)

has an inverse

$$D^{-1} = \frac{1}{\rho h} \int_0^t \int_S \left[ \sum_{i,j} \phi_{ij}(x,y) \phi_{ij}(\xi,\eta) \sin \frac{\omega_{ij}(t-\tau)}{\omega_{ij}} \right] \ldots d\rho d\tau dt ,$$  \hspace{1cm} (5.5)