Chapter 5

Physics-Based Models of Snow

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Abstract. This chapter is a review of current physics-based techniques for modeling snowmelt. Mixture theory is used to develop the equations of conservation of mass, momentum and energy for snow treated as a two-component, three-phase mixture of ice, water, water vapor, and air. The constitutive laws and boundary conditions required to complete a general snowmelt model are then described, with particular attention to the energy balance at the upper boundary of a snowpack.

1. Introduction

Modern physics-based snowmelt modeling began with Anderson (1976) who replaced the simple index methods then in use by an energy budget approach. Instead of calculating snowmelt from meteorological data (usually only air temperature) by simple linear empirical equations he calculated all the components of the energy input to the snow surface, the energy losses from the surface, and hence the residual energy available to melt the snow. Although Anderson used empirical equations to describe processes within the snow pack, i.e. heat and water flow, it soon became clear that the energy budget approach could be much improved by physics-based modeling of internal as well as boundary processes. The appropriate theory already existed in the hydrological literature. Colbeck realized that the equations of flow for soils could also be applied to snow since both were porous media. In a series of papers (Colbeck, 1972, 1974, 1977) he established the techniques for modeling water flow in wet (or 'ripe') snow. Meanwhile, other authors (Sulakvelidze, 1959; Yen, 1962; Navarre, 1977; Obled and Rosse, 1977; Palm and Tveitereid, 1979) were applying established equations for heat flow in porous media to the problem of modeling temperature variations in cold snow. Morris and Godfrey (1979) made the first attempt to construct a physics-based model which could be used for both cold and ripe snow and which took into account changes in the structure of the snow pack. Since then, similar models have been developed not only for hydrological purposes but also for avalanche prediction and glacier mass balance research (e.g. Morris, 1983; Akan, 1984; Greuell and Oerlemans, 1986).

Recently the mathematical basis of these snowmelt models has been critically examined by Kelly and his coworkers (Kelly et al., 1986; Kelly, 1987). Several interesting mathematical problems have arisen and have provoked a reexamination of the physics of the problem. This chapter will describe the 'state-of-the-art' snow melt model equations in detail, drawing attention to the assumptions which have been made about the physics and their consequences.

So far, physics-based models have only been written for 'unpolluted' snow, that is for a mixture of ice, water, water vapor, and air. However, it is now clear that a major application of
snowmelt modeling will be simulation of the preferential elution of impurities at the onset of snowmelt. For this reason the chapter contains rather more discussion of the effect of impurities than is strictly required. Further work is needed on the complex physical chemistry of concentrated ionic solutions in thin films before a working program for polluted snow can be written, but it is useful now to define the required mathematical structure of the model even if all the equations are not known.

2. Discussion

2.1. Mixture Theory

The basis for physics-based snowmelt models is the concept that snow can be regarded as a mixture containing two components: air; and water in up to three of its phases, ice, liquid water, and water vapor. Each volume element of snow is assumed to contain all the constituents dispersed within it. This approach has been used by many authors (e.g. Yen, 1962; Colbeck, 1972; Obled and Rosse, 1977; Navarre, 1977; Morris and Godfrey, 1979; Morris, 1983; Akan, 1984) although different degrees of simplification have been proposed. To date, a working 2-component, 3-phase snowmelt model has not been achieved but considerable progress towards defining the necessary equations has been made by Kelly (1987). In this chapter the model equations will be developed in one-dimension for ease of notation but it is not difficult to extend them to three-dimensions.

Let the independent variables defining the state of the bulk phases be the pressures, \( p_\alpha \), the temperatures, \( T_\alpha \), and the number of moles, \( n^\alpha_k \), per unit volume of snow and velocities, \( w^\alpha_k \), of each of the components. Let \( \alpha = 1 \) for the solid phase, \( \alpha = 2 \) for the liquid phase, and \( \alpha = 3 \) for the gas. The two components are water \((k = 1)\) and air \((k = 2)\). Similarly, let the independent variables defining the state of the interfaces be the surface tensions, \( \sigma_i \), the temperatures, \( T^i \), and the number of adsorbed moles per unit volume of snow, \( n^i_k \), and velocities, \( w^i_k \), of each component. Three types of interface are possible: \( i = 1 \) for an interface between solid and liquid phases, \( i = 2 \) for a liquid-gas interface, and \( i = 3 \) for a solid-gas interface.

2.1.1. The Conservation Equations

Conservation of Mass. Changes in the system with time can be described by the conservation equations for mass, energy, and momentum. Let \( N^\alpha_k \) be the number of moles of component \( k \) gained by bulk phase \( \alpha \) per unit time per unit volume of snow. Then conservation of mass gives:

\[
\frac{d(n^\alpha_k)}{dt} = \frac{\partial(n^\alpha_k)}{\partial t} + w^\alpha_k \frac{\partial(n^\alpha_k)}{\partial z} = N^\alpha_k
\]

where \( t \) is time and \( z \) the vertical direction.

A similar equation may be written for \( N^i_k \), the number of moles of component \( k \) gained by interface \( i \) per unit time per unit volume of snow. Assuming that there are no chemical reactions which transform component \( k \) into a different component, the number of moles of \( k \) per unit volume of snow is conserved:

\[
\sum_\alpha N^\alpha_k + \sum_i N^i_k = 0
\]