In this chapter we present several examples of codes. Each example is in fact a method to construct some family of codes, which (in some way or other) have rather good parameters. Since in many cases these families are predecessors of algebraic-geometric codes, we try to choose constructions that are easy to generalize in that direction.

Section 1.2.1 is devoted to the example most important for us, to Reed-Solomon codes, which - as we shall see in Part 3 - are just algebraic-geometric codes of genus zero. We discuss them in detail, calculate the spectra, and give a decoding algorithm. In Section 1.2.2 we briefly discuss other interesting families of codes. Section 1.2.3 is devoted to a number of rather simple constructions which produce new codes starting with some codes we already know.
1.2.1. Codes of genus zero

Recall that a code of genus zero (or an MDS-code) is an $[n,k,d]_q$-code $C$ such that $k + d = n + 1$; this yields $(n - k) + d^l = n + 1$, i.e. $C^l$ is also an MDS-code (see Exercise 1.1.18).

Trivial codes. For any $n$ there are three simple $q$-ary codes which it is naturally to call trivial. These are:

- $[n,n,1]_q$-code $C_1 = \mathbb{F}_q^n$,
- $[n,n-1,2]_q$-code $C_2 = \{ (v_1, \ldots, v_n) \in \mathbb{F}_q^n | \sum v_i = 0 \}$ (called the parity-check code), and
- $[n,1,n]_q$-code $C_3 = \{ v = (\alpha, \ldots, \alpha) \in \mathbb{F}_q^n \}, \alpha \in \mathbb{F}_q$ (called the repetition code).

Now we are passing to a more conceptual construction.

Reed-Solomon codes. Let $\mathcal{P} = \{ P_1, \ldots, P_n \} \subseteq \mathbb{F}_q^n$ be a subset of cardinality $n$. Consider a linear space $L(a)$ of all polynomials in one variable of degree at most $a$ with coefficients in $\mathbb{F}_q$; $\dim L(a) = a + 1$. For $n > a$ a non-zero polynomial $f(x) \in L(a)$ cannot vanish at all points of $\mathcal{P}$, moreover, it has at least $(n - a)$ non-zero values at points of $\mathcal{P}$. The "evaluation" map

$$ Ev_{\mathcal{P}} : L(a) \longrightarrow \mathbb{F}_q^n $$

$$ Ev_{\mathcal{P}} : f \longmapsto (f(P_1), \ldots, f(P_n)) $$

is injective for $n > a$ and its image $C$ is an