Chapter 14

Population Uncertainty and Equilibrium Selection: a Maximum Likelihood Approach

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14.1 Introduction

In games with incomplete information (Harsanyi, 1967–1968) as usually studied by game theorists, the characteristics or types of the participating players are possibly subject to uncertainty, but the number of players is common knowledge. Recently, however, Myerson (1998a, 1998b, 1998c, 2000) and Milchtaich (1997) proposed models for situations—like elections and auctions—in which it may be inappropriate to assume common knowledge of the player set. In such games with population uncertainty, the set of actual players and their preferences are determined by chance according to a commonly known probability measure (a Poisson distribution in Myerson’s work, a point process in Milchtaich’s paper) and players have to choose their strategies before the player set is revealed.

After the introduction of the maximum likelihood principle by R.A. Fisher in the early 1920’s (see Aldrich, 1997, for an interesting historical account), the method of selection on the basis of what is most likely to
be right has gained tremendous popularity in the field of science dealing with uncertainty. Gilboa and Schmeidler (1999) recently provided an axiomatic foundation for rankings according to the likelihood function.

A first major topic of this chapter is the introduction of a general class of games with population uncertainty, including the Poisson games of Myerson (1998c) and the random-player games of Milchtaich (1997). In line with the maximum likelihood principle, the present chapter stresses those strategy profiles in a game with population uncertainty that are most likely to yield an equilibrium in the game selected by chance. Maximum likelihood equilibria were introduced in Borm et al. (1995a) in a class of Bayesian games.

The $\sigma$-algebra underlying the chance event that selects the actual game to be played may be too coarse to make the event in which a specific strategy profile yields an equilibrium measurable. A common mathematical approach (also used in a decision theoretic framework; cf. Fagin and Halpern, 1991) to assign probabilities to such events is to use the inner measure induced by the probability measure. Roughly, the inner measure of an event $E$ is the probability of the largest measurable event included in $E$.

Under mild topological restrictions, an existence result for maximum likelihood equilibria is derived. Since the result establishes the existence of a maximum of the likelihood function, it differs significantly from standard equilibrium existence results that usually rely on a fixed point argument.

The use of inner measures is intuitively appealing and avoids measurability conditions. Still, the measurability issue is briefly addressed and it is shown that under reasonable restrictions the inner measure can be replaced by the probability measure underlying the chance event that selects the actual game. Moreover, it is shown that games with population uncertainty can be used to model situations in which, in addition to the set of players and their preferences, also the set of feasible actions of each player is subject to uncertainty. This captures a common problem in decision making, namely the situation in which decision makers have to plan or decide on their course of action, while still uncertain about contingencies that may make their choices impossible to implement.

A second major topic of this chapter is the use of maximum likelihood equilibria as an equilibrium selection device for finite strategic games. A finite strategic game can be perturbed by assuming that with a commonly known probability distribution there are trembles in the payoff