Bayesians typically hold that rationality mandates that we conform our degrees of belief to the axioms of the probability calculus. Starting with Ramsey (1926) and de Finetti (1937), they have sought to defend their position by arguing that we are susceptible to “Dutch books”, i.e. bets that ensure a negative net pay-off in every possible future, just in case our degrees of belief violate the axioms of probability. It is by now widely recognized that such Dutch book arguments rest on premises that are – to say the least – fairly problematic. For instance, these arguments assume that we can tell an individual’s degrees of belief from her willingness to engage in certain bets. More exactly, they assume that we can identify, for any proposition \( A \), an individual’s degree of belief in \( A \) with her fair betting quotient for \( A \), that is, with the highest/lowest price, in units utile, at which she is willing to place/accept a bet on \( A \) that pays one utile if \( A \) is true (and nothing otherwise). While this assumption may have seemed plausible at the time when operationalism and behaviourism ruled the day (when Ramsey and de Finetti devised their Dutch book arguments), to the modern eye it looks rather suspect. Schick (1986), Bacchus, Kyburg and Thalos (1990) and Waidacher (1997), among others, have pointed to further contentious assumptions underlying Dutch book arguments.

A different approach to justifying the Bayesian theory thus seems more than welcome. Howson (2003) promises to offer just that. His aim is to defend the Bayesian theory as a logic, concerned with consistent reasoning, and not as being part of a broader theory concerned with rational (betting) behaviour. In this note I will argue that, although I can see nothing wrong with thinking about the Bayesian theory as a logic, it is far from clear how its status as a logic can contribute to the theory’s justificational status. My main point of critique will be that the fact that Bayesian logic can be proved to be sound and complete, as Howson demonstrates, is by itself of very little significance, contrary to what Howson seems to believe.

According to Howson (2000, p. 127), a theory has logical status exactly if it satisfies three conditions. The first two are that the theory must be about relations between statements and that there must be no restrictions as
regards the subject matter of these statements, respectively. The third, and in the context of this note most important condition, says that the theory “should incorporate a semantic notion of consistency which can be shown to be extensionally equivalent to one characterizable purely syntactically” (ibid.). That probability theory satisfies the first two conditions is immediate. That it also satisfies the third is not immediate, but Howson shows that it does nonetheless by proving a soundness and completeness theorem for probability theory. More specifically, he shows that, given some language L, an assignment of fair betting quotients to a set of propositions expressible in L has a model (in a particular sense; see below) exactly if that assignment conforms to the probability axioms (Howson 2000, pp. 130ff). This soundness and completeness theorem is, from a purely technical point of view, certainly an important result. However, I fail to see how it can offer a justification of the probability axioms. That probability theory is sound and complete in the sense meant by Howson certainly does not count against that theory, but that, in my view, is about all we can conclude.

To explain this, let me first briefly recall that there is an issue of justification not only for inductive logic but also for deductive logic. There exists a welter of rival (deductive) logics, all seemingly purporting to formalize what we might call the laws of truth. The question which of these is the true logic has been, and still is, hotly debated in philosophy. For present concerns just notice that to defend a particular logic by claiming that it is sound and complete would be entirely disingenuous. Although not all logics are sound and complete, many of them are. This is not necessarily to say that the justification of a particular logic as being the correct logic is wholly detached from considerations that have to do with soundness and/or completeness. As will be remembered, to say of a logic that it is sound, or that it is complete, is to say that it is sound/complete with respect to a given semantics. To be very precise, then, we should say that a logic is sound with respect to semantics S just in case reasoning according to the rules of that logic is X-preserving, where X is the central semantic concept of S. Similarly, we should say that it is complete with respect to S just in case any sentence that has property X, on the condition that all elements of a certain (possibly empty) collection of sentences have X, can be derived from the sentences in that collection by means of the rules of the logic. And depending on what the central semantic concept is, a soundness cum completeness proof may be of more or less significance (even if, as I suspect, it may never be wholly decisive as a justification of a given logic).