Chapter 8

FIRST ORDER MACROSCOPIC TRAFFIC FLOW MODELS FOR NETWORKS IN THE CONTEXT OF DYNAMIC ASSIGNMENT

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Abstract The purpose of the paper is to adapt the classical LWR (Lighthill-Whitham-Richards) model, in its continuous version, to networks, in the context of dynamic assignment. This implies several specific adaptations of the basic model: introduction of partial flows, possibly inhomogeneous flows on links, and intersection modeling. The latter proves particularly difficult, and we discuss three different modeling approaches: extended versus pointwise intersection models, and flow maximization. We show that all three types of models are actually closely related, and compatible with the link flow models. The concepts of local traffic supply and demand prove to be essential, both for link and for intersection modeling. A brief comparison with experimental merge data gives some support to the phenomenological models introduced in the paper.

1. Introduction

Modeling traffic flow for dynamic assignment is a difficult task, because a compromise has to be struck between conflicting imperatives. The model should be realistic, accommodate partial flows, be suitable for large networks and at the same time be very simple in order to be compatible with intensive numerical computation. The general trend in traffic models for dynamic assignment has been one of increasing sophistication, starting with the simple exit function models [23], moving on to travel time function models [7], [25], point queue models [11] and finally macroscopic models ([20], [12], [6], [27], [2], [1]).
These macroscopic models rely on the discretization of their basic equations, and on heuristics for the intersection description. It is therefore very difficult to obtain for instance optimality conditions for assignment problems with such macroscopic models. This has only been attempted in the simplest of cases [9].

The object of the proposed paper is, in the case of the simpler LWR (Lighthill-Whitham-Richards [21], [26]) model, to provide a general methodology for a rigorous description of networks in the context of dynamical assignment. Our choice of the LWR model is justified by: its simplicity, the availability of nontrivial analytical solutions, the absence of inconsistent features ([4]).

The outline of the paper is the following. After recalling the basic LWR model and introducing the local traffic supply and demand concept, we describe boundary conditions and link models for partial flows, i.e. flows disaggregated according to some attribute. We then proceed to intersection modeling, beginning with the so-called zone model in which the physical extension of the intersection is taken into account. By neglecting the dimension of the intersection, we can deduce pointwise intersection models. These can be shown to result also from some flow-maximization principle. We conclude with a brief comparison between model prediction and experimental data in the case of a simple merge.

2. The basic model LWR model for a link

The basic LWR model can be written as:

\[
\begin{align*}
\frac{\partial K}{\partial t} + \frac{\partial Q}{\partial x} &= 0 \quad \text{conservation equation} \\
Q &= KV \quad \text{definition of } V \\
V &= V_e(K, x) \quad \text{behavioral equation.}
\end{align*}
\]

or simpler as:

\[
\frac{\partial K}{\partial t} + \frac{\partial}{\partial x} Q_e(K, x) = 0,
\]

with \( Q \) the flow, \( K \) the density, \( V \) the speed, \( Q_e \) and \( V_e \) the equilibrium flow-resp. speed-density relationships \((Q_e(K, x) \overset{\text{def}}{=} KV_e(K, x)) \). As usually, \( x \) and \( t \) denote position and time. The aspect of \( Q_e \) is the following:

![Diagram of Q_e]