CHAPTER 7.
BEM FOR CRACK DYNAMICS

M.H. ALIABADI
Department of Engineering
Queen Mary, University of London, E1 4NS, UK

Abstract
In this Chapter the modern application of the boundary element method to crack problems in dynamic fracture mechanics is reviewed. Dual boundary formulations are presented for the time domain, Laplace transform and dual reciprocity methods. The three approaches are applied to mixed mode two-dimensional and three-dimensional crack problems.

Keywords: Boundary element method, fracture, crack propagation, dynamic stress intensity factor

1. Introduction
The aim of dynamic fracture mechanics is to analyze the growth, arrest and branching of moving cracks in structures subjected to dynamic loads. The stress field in the vicinity of the crack is usually characterized by dynamic stress intensity factors (DSIF) which are generally functions of time. Structures with arbitrary shape and time-dependent boundary conditions need to be analyzed by numerical methods. One of the earliest studies of the transient problem can be found in the paper by Baker (1962). Later Achenbach and Nuismer (1971) extended Baker’s work to include incident waves of arbitrary stress profile. A review of state of the art techniques in computational dynamic fracture mechanics can be found in Aliabadi (1994) where different modelling approaches such as the Finite Element Method, the Boundary Element Method and the Finite Volume Method are described. The Boundary Element Method (BEM) of analysis is better suited to crack problems than the more established Finite Element Method.
because the crack and crack propagation modelling are simpler. Some reviews of boundary element methods for the numerical solution of elastodynamic problems are given by Beskos (1998) and Dominguez and Gallego (1992). Solutions in elastodynamics using the BEM are usually obtained by either the time domain method, Laplace or Fourier transforms or the dual reciprocity method.

The time domain method was used by Nishimura, Guo and Kobayashi (1987) to solve crack problems. The boundary integral equations in that formulation contain hypersingular integrals, which were regularized using integration by parts twice. Constant spatial and linear temporal shape functions were used for the approximations. The method was applied for stationary and growing straight cracks in two-dimensional, and plane cracks in three-dimensional infinite domains. Zhang and Gross (1997) used the two-state conservation integral of elastodynamics, which leads to non-hypersingular traction boundary integral equations. The unknown quantities in that approach are the crack opening displacements and their derivatives. Similar time and space discretizations were used. The method was applied to penny-shaped and square cracks in infinite domains. Hirose (1991) used the formulation based on the traction equation. Piecewise linear temporal functions were used and the displacements of the crack were interpolated using the analytical solution of the static problem. The method was applied for both stationary and growing penny-shaped cracks.

The Laplace transform method was used by Sládek and Sládek (1986) who analyzed a penny-shaped crack in an infinite elastic body under a harmonic and an impact load using the traction equation and the displacement discontinuity method. A polar coordinate system was assumed and a linear variation of displacements along the radius. They used the displacement equation to analyze symmetric problems, which require discretization of a part of the body only, a rectangular plate with edge cracks and a thick walled tube with radial cracks.

Application of the indirect displacement discontinuity method to dynamic crack problems was developed by Wen, Aliabadi and Rooke (1995a, 1996a, 1998ab).

Recently Fedelinski, Aliabadi and Rooke (1993, 1994, 1995, 1996ab, 1997), Wen, Aliabadi and Rooke (1998c) and Wen, Aliabadi and Young (1999ac) have developed time domain method, Laplace transform method and dual reciprocity method in dual boundary element analysis for two-dimensional and three-dimensional dynamic fracture mechanics problems respectively. By using the displacement equation and traction equation on