Optimal Inspection in an Assembly System

In contrast to Chapter 7, where the problem is to coordinate the inspections at two stages in series, here we switch to a parallel configuration—an assembly system—and focus on how to coordinate the inspections at component subassemblies.

Specifically, suppose each end-product consists of two different component subassemblies and that quality control of the components is carried out before the final assembly. The problem is to develop an inspection policy on the two component batches so that the total cost is minimized. Although the optimal policy can be derived from solving a dynamic programming problem, there is no guarantee that this will result in a simple control policy, such as the threshold-type sequential inspection policies that were proven optimal in earlier chapters (in particular, Chapters 3 and 6). In fact, as the inspection decisions for the two components are interleaved together, the optimal policy in general will switch back and forth between the two components, destroying any threshold structure. Our focus here is on a class of easily implementable policies that have a simple “single-switch” structure. We show that this class of policies is optimal for a special case of the original problem when one of the two components has a constant defective rate. Furthermore, we illustrate through numerical examples that such policies have near-optimal performance when applied to the original problem.

In §8.1 we describe the model in detail and derive certain basic properties of it. A dynamic programming formulation is presented in §8.2. In §8.3 we identify an optimal policy that has a threshold structure for the case when one of the components has a constant defective rate. Based on this result, in
§8.4 we develop a heuristic inspection policy for the original problem, and illustrate its near-optimal performance through two numerical examples.

8.1 A Two-Component Assembly Model

Suppose two components, termed component 1 and component 2, are produced in batches with \( N \) units per batch for both components. They are then assembled into \( N \) end-products, which, in turn, are supplied to customers under some types of warranty (or service contract). Suppose the defective rates of the two components are \( \Theta_1 \) and \( \Theta_2 \), respectively. Here, \( \Theta_i \in [0, 1] \) is itself a random variable with a known distribution, representing the defective rate of component \( i \). This works as follows: \( \Theta_i \) is first sampled from its distribution. Suppose the sample value is \( \theta_i \); then each unit in the batch is defective with probability \( \theta_i \). The distribution of \( \Theta_i \), which is common for all the units in the batch, naturally captures the statistical dependence among the units. In this paper \( \Theta_1 \) and \( \Theta_2 \) are supposed to be independent, and we assume that the assembly procedure itself does not produce defective products, i.e. as long as the two components are both nondefective, they will be assembled into a nondefective end-product.

For \( i = 1, 2 \), suppose the lifetime of a defective component \( i \) is \( Y_i \), and a nondefective component \( i \) has a lifetime \( X_i \). Assume \( X_i \geq_{st} Y_i \) and the lifetimes of the components are independent of one another. If the lifetimes of the two components are \( Z_1 \) and \( Z_2 \), respectively, then the lifetime of the end-product is assumed to be \( Z_1 \land Z_2 := \min(Z_1, Z_2) \). Let \( Z_i(\theta_i) \) denote the random variable equal in distribution to \( Y_i \) (resp. \( X_i \)) with probability \( \theta_i \) (resp. \( 1 - \theta_i \)).

Defectives of both components can be detected by inspection. Suppose the inspection is perfect (i.e., a component is identified as defective if and only if it is defective), and the per-unit inspection cost is \( C_I^{(1)} \) and \( C_I^{(2)} \), respectively, for the two components. A defective unit can be corrected via repair or rework (and become a nondefective unit) at a per-unit cost of \( C_R^{(1)} \) or \( C_R^{(2)} \), respectively, for the two components.

Similar to Chapter 3, we will first consider a cumulative warranty cost function of the end-products. Specifically, if the total lifetime of the \( N \) assembled units is \( t \), then the warranty cost is \( C(t) \), with \( C(t) \) assumed to be a convex and decreasing function of \( t \). When \( C(t) \) is an additive function, we have the individual warranty case in which the warranty applies to each individual unit. To provide adequate incentive to the repair of any defective unit, we assume

\[
\begin{align*}
EC\left( \sum_{j=1}^{N-1} X_{1j} \land X_{2j} + Y_1 \land Y_2 \right) - EC\left( \sum_{j=1}^{N-1} X_{1j} \land X_{2j} + X_1 \land Y_2 \right) & \geq C_R^{(1)}.
\end{align*}
\]  

(8.1)