Chapter 17

MULTIOBJECTIVE PROGRAMMING

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Abstract We present our view of the state of the art in multiobjective programming. After an introduction we formulate the multiobjective program (MOP) and define the most important solution concepts. We then summarize the properties of efficient and nondominated sets. In Section 4 optimality conditions are reviewed. The main part of the chapter consists of Sections 5 and 6 that deal with solution techniques for MOPs and approximation of efficient and nondominated sets. In Section 7 we discuss specially-structured problems including linear and discrete MOPs as well as selected nonlinear MOPs. In Section 8 we present our perspective on future research directions.

Keywords: Multiobjective programming, efficient solution, nondominated solution, scalarization, approximation.
1. Introduction

Multiobjective programming is a part of mathematical programming dealing with decision problems characterized by multiple and conflicting objective functions that are to be optimized over a feasible set of decisions. Such problems, referred to as multiobjective programs (MOPs), are commonly encountered in many areas of human activity including engineering, management, and others. Throughout the chapter we understand multiobjective programming as pertaining to situations where feasible alternatives are available implicitly, through constraints in the form of mathematical functions. An optimization problem (typically a mathematical program) has to be solved to explicitly find the alternatives. Decision problems with multiple criteria and explicitly available alternatives are treated within multicriteria decision analysis (MCDA). This view constitutes the difference between multiobjective programming and multicriteria decision analysis (MCDA) which complement each other within multicriteria decision making (MCDM).

In the last fifty years, a great deal of theoretical, methodological and applied studies have been undertaken in the area of multiobjective programming. This chapter presents a review of the theory and methodology of MOPs in finite dimensions. The content of the review is based on the understanding that the primary (although not necessarily the ultimate) goal of multiobjective programming is to seek solutions of MOPs. Consequently, methods suitable for finding these solutions are considered the most fundamental tools for dealing with MOPs and therefore given special attention. The selection of a preferred solution of the MOP performed by the decision maker can be considered the ultimate goal of MCDM. However, the modelling of decision maker preferences is outside the scope of this chapter and belongs to the domain of MCDA.

In Sections 2, 3, and 4 we review theoretical foundations of multiobjective programming. In Section 2 we define MOPs and relevant solution concepts. Sections 3 and 4 contain a summary of properties of the solution sets and conditions for efficiency, respectively. The subsequent sections focus on methodological aspects of multiobjective programming. In Sections 5 and 6, numerous methods for generating individual elements or subsets of the solution sets are collected. In these sections we present scalarization, nonscalarizing and approximation methods. Specially structured problems, including linear, combinatorial and nonlinear MOPs, are discussed in Section 7. The chapter is concluded in Section 8 with our view of current and future research directions.

We point out that the results are not always presented chronologically but rather with respect to the order implied by the content of this chapter and with respect to their level of generality.

The following notation is used. Let $\mathbb{R}^p$ be the Euclidean vector space and $y, y' \in \mathbb{R}^p$. 