Chapter 5

Discrete Random Variables

5.1 Introduction

Having been introduced to the basic probabilistic concepts in Chapters 3 and 4, we now begin their application to solving problems of interest. To do so we define the random variable. It will be seen to be a function, also called a mapping, of the outcomes of a random experiment to the set of real numbers. With this association we are able to use the real number description to quantify items of interest. In this chapter we describe the discrete random variable, which is one that takes on a finite or countably infinite number of values. Later we will extend the definition to a random variable that takes on a continuum of values, the continuous random variable. The mathematics associated with a discrete random variable are inherently simpler and so conceptualization is facilitated by first concentrating on the discrete problem. The reader has already been introduced to the concept of a random variable in Chapter 2 in an informal way and hence may wish to review the computer simulation methodology described therein.

5.2 Summary

The random variable, which is a mapping from the sample space into the set of real numbers, is formally discussed and illustrated in Section 5.3. In Section 5.4 the probability of a random variable taking on its possible values is given by (5.2). Next the probability mass function is defined by (5.3). Some important probability mass functions are summarized in Section 5.5. They include the Bernoulli (5.5), the binomial (5.6), the geometric (5.7), and the Poisson (5.8). The binomial probability mass function can be approximated by the Poisson as shown in Figure 5.8 if \( M \to \infty \) and \( p \to 0 \), with \( Mp \) remaining constant. This motivates the use of the Poisson probability mass function for traffic modeling. If a random variable is transformed to a new one via a mapping, then the new random variable has a probability mass function given by (5.9). Next the cumulative distribution function is introduced and
is given by (5.10). It can be used as an equivalent description for the probability of a discrete random variable. Its properties are summarized in Section 5.8. The computer simulation of discrete random variables is revisited in Section 5.9 with the estimate of the probability mass function and the cumulative distribution function given by (5.14) and (5.15), (5.16), respectively. Finally, the application of the Poisson probability model to determining the resources required to service customers is described in Section 5.10.

5.3 Definition of Discrete Random Variable

We have previously used a coin toss and a die toss as examples of a random experiment. In the case of a die toss the outcomes comprised the sample space $S = \{1, 2, 3, 4, 5, 6\}$. This was because each face of a die has a dot pattern consisting of 1, 2, 3, 4, 5, or 6 dots. A natural description of the outcome of a die toss is therefore the number of dots observed on the face that appears upward. In effect, we have mapped the dot pattern into the number of dots in describing the outcome. This type of experiment is called a numerically valued random phenomenon since the basic output is a real number. In the case of a coin toss the outcomes comprise the nonnumerical sample space $S = \{\text{head, tail}\}$. We have, however, at times replaced the sample space by one consisting only of real numbers such as $S_X = \{0, 1\}$, where a head is mapped into a 1 and a tail is mapped into a 0. This mapping is shown in Figure 5.1. For many applications this is a convenient mapping. For example, in a succession of $M$ coin tosses, we might be interested in the total number of heads observed. With the defined mapping of

$$X(s_i) = \begin{cases} 
0 & s_1 = \text{tail} \\
1 & s_2 = \text{head} 
\end{cases}$$

![Figure 5.1: Mapping of the outcome of a coin toss into the set of real numbers.](image)