Abstract. Building on Michael Otte's insights regarding the roles of icon, index, and symbol in mathematical signification, definitions of these categories of representation are explored in terms of metaphors and metonymies. A nested model of signs, based on Peirce's triadic formulation, is described, along with his trichotomic distinction among interpretants that are intentional, effectual, and communicational (leading to the **commens**). The theoretical argument and its utility is illustrated in terms of an episode of creating a proof in a college geometry class. The significance of the theoretical notions for creativity in mathematics is seen to reside in metaphorical and metonymical processes.

**Key words:** commens, icon, index, metaphor, metonymy, representamen, symbol, universals

The reasoning of mathematicians will be found to turn chiefly upon the use of likenesses, which are the very hinges of the gates of their science. The utility of likenesses to mathematicians consists in their suggesting, in a very precise way, new aspects of supposed states of things. (Peirce 1998, 6)

1. HOW AND WHY IS SEMIOTICS USEFUL IN MATHEMATICS EDUCATION?

After all, some of the originators of theories of semiotics were linguists. Ferdinand de Saussure's (1959) book, *Course in General Linguistics*, is a seminal work in this area. And Charles Sanders Peirce, himself fluent in Latin, Greek, and several other languages, makes it abundantly apparent in his writings (e.g., 1998, Vol. 2) that semiotics undergirds and illuminates the study of languages and their structure. Why, then, is semiotics, defined as the study of semiosis (activity with signs), useful to mathematics educators? A hint of an answer to this question is given in the initial quotation from Peirce, and in this chapter I analyze semiotic aspects of metaphor and metonymy in particular, showing the relevance of "the use of likenesses" for deductive thinking and problem solving in the learning of mathematics. In a triadic model of nested signs based on the formulation of Peirce, the categorization of signs as iconic, indexical, or symbolic relates to the uses of metaphor and metonymy in semiosis. Informed by the insights of Michael Otte, I have found these constructs to be powerful lenses in my research, both on ways of connecting home activities of students with formal mathematical concepts in school and college (Presmeg 2002), and in understanding the ways that signs support learning of mathematics at all levels. After an initial description of this triadic nested model of signs and their uses,
I analyze in more depth how metaphor and metonymy are implicated in the model, concluding with an analysis of an episode in a college geometry course.

2. A TRIADIC NESTED MODEL OF SEMIOSIS

Implicit in Peirce’s triadic model of semiosis is a nesting effect. However, his writing is dense with ideas, some of which are explicated in careful detail, while others are barely sketched and further elaboration is left to the reader (e. g., his ten trichotomies, 1998, 481-491). Thus it is useful to consider in detail how this nesting of signs occurs. In this chapter I shall refer mainly to the trichotomy that by his own account he used most often, namely, a trichotomy designating three kinds of signs, which he called icons, indices, and symbols. But first it is necessary to say a few words about terminology. In the previous two sentences I have used the word signs in two different ways, and this is what Peirce does too, in different parts of his writings. To avoid confusion I shall use the word sign to refer to the totality of object, representamen, and interpretant (the former usage above – these signs are nested), and not to the representamen specifically (the latter usage). Thus when Peirce designates icons, indices, and symbols as three kinds of “signs”, he is referring to three ways in which the representamen may stand in relation to its object; and the interpretant is then the result of reflection on this relationship – and thus is indirectly implicated. If we take an example suggested by Peirce (1998) and elaborated by Whitson (1994, 1997), the falling barometer (representamen) suggests that it will rain (object), but an act of interpretation is involved, and the observer may decide to take an umbrella (interpretant). That the interpretant is in terms of an action – the taking of an umbrella – reminds us that Peirce was one of the founders of pragmatism; but this aspect of his writings will not be pursued in this chapter. “To define a sign we therefore need an object as well as an interpreter” (Otte 2001, 5).

Michael Otte, in his writings on semiotics (e. g., 2001), usually invokes the representamen as “sign” following this usage in most of Peirce’s work. As Otte (2001) remarked, Peirce (1998) defined a sign as anything that stands for something (called its object) in such a way as to generate another sign (its interpretant or meaning). This definition involves a double use of the term. The import of the definition seems to be that a sign is anything that stands for something else. In this case the interpretant (meaning) also stands for the relationship of the first two components. Although Otte’s usage is consistent with this definition of Peirce, I shall not follow it here, because I am particularly interested in clarifying and describing the totality of object, representamen, and interpretant as a sign that becomes reified (Sfard 1992) as a new object in a nesting process that could continue indefinitely (as is also implied by Peirce’s definition). A diagram casts light on the structure of the relationships.

Each of the rectangles in Figure 1 represents a sign consisting of the triad of object, representamen, and interpretant, corresponding roughly to signified, signifier, and a third interpreted component, respectively. This interpretant involves meaning making: it is the result of trying to make sense of the relationship of the other two components, the object and the representamen. It is important to note that