SEMISMOOTH NEWTON METHODS FOR SHAPE-PRESERVING INTERPOLATION, OPTION PRICE AND SEMI-INFINITE PROGRAMS

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Abstract: In this paper, we survey the development of semismooth Newton methods for solving the shape-preserving interpolation problem, the option price problem, and the semi-infinite programming problem.

Key words: Smoothness, Semismoothness, shape-preserving interpolation, option price problem, semi-infinite programming

1. INTRODUCTION

The semismooth Newton method was initiated in 1993 by Qi [31], Qi and Sun [37]. It has found applications in nonlinear complementarity problems [27,7,22,14,45,8,26], variational inequality problems [13,33], civil engineering problems [5] and data mining problems [16]. In [16], a semismooth Newton method was used for solving a 60 million variable support vector machine problem successfully. A survey on the semismooth Newton method can be found in [23]. Some developments after [23] can be found in [17,33,18]. The applications of semismooth Newton methods on nonlinear complementarity problems and variational inequality problems can be found in the recent book of Facchinei and Pang [15], where several hundreds of references on semismooth Newton methods were given.
Recently, semismooth Newton methods have been applied to the shape-preserving interpolation problem, the option price problem, and the semi-infinite programming problem. We survey these applications in this paper.

2. SEMISMOOTH NEWTON METHODS

Let \( G : \mathbb{R}^n \to \mathbb{R}^m \) be a locally Lipschitz function. By the Radamacher theorem, \( G \) is differentiable almost everywhere. Denote the set on which \( G \) is differentiable as \( D_G \). Clarke [6] defined the generalized Jacobians of \( G \) at \( x \) as

\[
\partial G(x) = \text{conv}\{ \lim \limits_{x' \to x, x' \in D_G} \nabla G(x') \},
\]

which is a nonempty compact convex set.

Suppose that \( F : \mathbb{R}^n \to \mathbb{R}^n \) is a locally Lipschitz function. We aim to solve

\[
F(x) = 0. \tag{1}
\]

A generalized Newton method is naturally available, i.e., given \( x^k \), if it is not a solution of (1), solve

\[
F(x^k) + V_k d = 0, \tag{2}
\]

where \( V_k \in \partial F(x^k) \). Let \( d^k \) be a solution of (2). Then we find \( x^{k+1} \) by:

\[
x^{k+1} = x^k + d^k. \tag{3}
\]

The subproblem (2) is a system of linear equations. So we may expect it is efficient. But counterexamples are available to show that the generalized Newton method (2-3) may be divergent if \( F \) is merely locally Lipschitz.

The superlinear and quadratic convergence of the generalized Newton method (2-3) can be established under the condition of semismoothness and strong semismoothness.

The concept of semismoothness for vector valued functions [37] is as follows.

**Definition 2.1.** Let \( G : \mathbb{R}^n \to \mathbb{R}^m \) be a locally Lipschitz continuous function. Then \( G \) is said to be semismooth at \( x \in \mathbb{R}^n \) if for any \( h \in \mathbb{R}^n \) the limit