INTRODUCTION TO QUANTUM FIELDS IN CURVED SPACETIME AND THE HAWKING EFFECT

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Abstract
These notes introduce the subject of quantum field theory in curved spacetime and some of its applications and the questions they raise. Topics include particle creation in time-dependent metrics, quantum origin of primordial perturbations, Hawking effect, the trans-Planckian question, and Hawking radiation on a lattice.

1. Introduction

Quantum gravity remains an outstanding problem of fundamental physics. The bottom line is we don’t even know the nature of the system that should be quantized. The spacetime metric may well be just a collective description of some more basic stuff. The fact[1] that the semi-classical Einstein equation can be derived by demanding that the first law of thermodynamics hold for local causal horizons, assuming the proportionality of entropy and area, leads one to suspect that the metric is only meaningful in the thermodynamic limit of something else. This led me at first to suggest that the metric shouldn’t be quantized at all. However I think this is wrong. Condensed matter physics abounds with examples of collective modes that become meaningless at short length scales, and which are nevertheless accurately treated as quantum fields within the appropriate domain. (Consider for example the sound field in a Bose-Einstein condensate of atoms, which loses meaning at scales below the so-called “healing length”, which is still several orders of magnitude longer than the atomic size of the fundamental constituents.) Similarly, there exists a perfectly good perturbative approach to quantum gravity in the framework of low energy effective field theory[2]. However, this is not regarded as a solution to the problem of quantum gravity, since the most pressing questions are non-perturbative in nature: the nature and fate of spacetime singularities, the fate of
Cauchy horizons, the nature of the microstates counted by black hole entropy, and the possible unification of gravity with other interactions.

At a shallower level, the perturbative approach of effective field theory is nevertheless relevant both for its indications about the deeper questions and for its application to physics phenomena in their own right. It leads in particular to the subject of quantum field theory in curved spacetime backgrounds, and the “back-reaction” of the quantum fields on such backgrounds. Some of the most prominent of these applications are the Hawking radiation by black holes, primordial density perturbations, and early universe phase transitions. It also fits into the larger category of quantum field theory (qft) in inhomogeneous and/or time-dependent backgrounds of other fields or matter media, and is also intimately tied to non-inertial effects in flat space qft such as the Unruh effect. The pertubative approach is also known as “semi-classical quantum gravity”, which refers to the setting where there is a well-defined classical background geometry about which the quantum fluctuations are occurring.

The present notes are an introduction to some of the essentials and phenomena of quantum field theory in curved spacetime. Familiarity with quantum mechanics and general relativity are assumed. Where computational steps are omitted I expect that the reader can fill these in as an exercise.

Given the importance of the subject, it is curious that there are not very many books dedicated to it. The standard reference by Birrell and Davies[3] was published twenty years ago, and another monograph by Grib, Mamaev, and Mostapanenko[4], half of which addresses strong background field effects in flat spacetime, was published two years earlier originally in Russian and then in English ten years ago. Two books with a somewhat more limited scope focusing on fundamentals with a mathematically rigorous point of view are those by Fulling[5] and Wald[6]. This year DeWitt[7] published a comprehensive two volume treatise with a much wider scope but including much material on quantum fields in curved spacetime. A number of review articles (see e.g. [8, 176, 10, 11, 12, 13]) and many shorter introductory lecture notes (see e.g. [14, 15, 16]) are also available. For more information on topics not explicitly referenced in the text of these notes the above references should be consulted.

In these notes the units are chosen with \( c = 1 \) but \( \hbar \) and \( G \) are kept explicit. The spacetime signature is \( (+-++) \). Please send corrections if you find things here that are wrong.

2. Planck length and black hole thermodynamics

Thanks to a scale separation it is useful to distinguish quantum field theory in a curved background spacetime (qftcs) from true quantum gravity (qg). Before launching into the qftcs formalism, it seems worthwhile to have a quick look at some of the interesting issues that qftcs is concerned with.