Chapter 12

CONTINUOUS MIN-MAX APPROACH FOR SINGLE PERIOD PORTFOLIO SELECTION PROBLEM

Nalan Gülpınar
Berç Rustem

Abstract In this chapter, we introduce continuous min-max approach for single period portfolio selection problem. The min-max optimization is performed over various single-period scenarios of risk and a return range, relative to benchmark. The optimal investment strategy is obtained using robust worst-case analysis. This evaluates the portfolio corresponding to the best performance, simultaneously with the worst-case. Therefore, the resulting strategy is robust in that it has the best lower bound performance which can only improve if any scenario, other than the worst-case, is realized.

1. Introduction

In financial portfolio management, the maximization of return for a level of risk is the accepted approach to decision making. A classical example is the single-period mean-variance optimization model in which expected portfolio return is maximized and risk measured by the variance of portfolio return is minimized Markowitz (1952). The mean-variance framework is based on a single forecast of return and risk. In reality, however, it is often difficult or impossible to rely on a single forecast. There are different rival risk and return estimates, or scenarios.

The inaccuracy in forecasting can be addressed through the specification of rival scenarios. These are used with forecast pooling using stochastic programming; for example see Kall (1994); Lawrence, Edmunson and O’Connor (1986); Makridakis and Winkler (1983). Robust pooling using min-max has been introduced by Rustem, Becker and Marty (2000) and Rustem and Howe (2002). A min-max algorithm for sto-
chastic programs based on bundle method is discussed by Breton and Hachem (1995). This is closer to the multi-stage model in Gülpinar and Rustem (2004) where a discrete min-max model is considered.

Min-max optimization is more robust to the realization of worst-case scenarios than considering a single scenario or an arbitrary pooling of scenarios. It is suitable for situations which need protection against risk of adopting the investment strategy based on the wrong scenario. There are two min-max models; discrete and continuous. The discrete min-max approach determines the optimal investment strategy in view of all specified discrete rival scenarios simultaneously, rather than any single scenario. Its disadvantage is that it requires the specification of a number of discrete scenarios. An alternative approach that addresses the specification of the return forecast in terms of a range given by upper and lower bounds is the continuous min-max. The continuous min-max strategy provides a guaranteed optimal performance in view of continuum of scenarios varying between upper and lower bounds. Thus, there are an infinite number of future scenarios in the continuous min-max framework.

In this chapter, we present a continuous min-max model for robust portfolio optimization based on worst-case analysis. The classical Markowitz framework is extended to the continuous min-max with upper and lower bounds on the return scenarios and various rival risk scenarios. The min-max model integrates benchmark relative computations in view of scalable (not fixed) transaction costs. Robustness arises from the non-inferiority of the worst-case optimal (min-max) strategy. We use the model for investment problems and evaluate the ex-ante performance of the strategy.

The rest of the chapter is organized as follows. In Section 2, we describe the mean-variance optimization model. The discrete and continuous min-max models are introduced in Section 3. The computational results are presented in Section 4.

**Notation**

A full description of our notation is given in Table 12.1. All quantities in boldface represent vectors in $\mathbb{R}^n$ unless otherwise noted. The transpose of a vector or matrix is denoted with the symbol $'$. 

2. Single period mean-variance optimization

The single period mean-variance optimization model considers a portfolio of $n$ assets defined in terms of a set of weights $w_i$ for $i = 1, \ldots, n,$