Chapter 2

SHORTEST PATH PROBLEMS WITH RESOURCE CONSTRAINTS

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Abstract

In most vehicle routing and crew scheduling applications solved by column generation, the subproblem corresponds to a shortest path problem with resource constraints (SPPRC) or one of its variants.

This chapter proposes a classification and a generic formulation for the SPPRCs, briefly discusses complex modeling issues involving resources, and presents the most commonly used SPPRC solution methods. First and foremost, it provides a comprehensive survey on the subject.

1. Introduction

For more than two decades, column generation (also known as branch-and-price when embedded in a branch-and-bound framework) has been successful at solving a wide variety of vehicle routing and crew scheduling problems (see e.g. Desrosiers et al., 1995; Barnhart et al., 1998; Desaulniers et al., 1998), and most chapters in this book). In most of these applications, the master problem of the column generation method is a (possibly generalized) set partitioning or set covering problem with side constraints, where most of the variables, if not all, are associated with vehicle routes or crew schedules. These route and schedule variables are generated by one or several subproblems, each of them corresponding to a shortest path problem with resource constraints (SPPRC) or one of its variants. The SPPRC has contributed to the success of the column generation method for this class of problems for three main reasons. Firstly, through its resource constraints, it constitutes a flexible tool for modeling complex cost structures for an individual route or schedule, as well as a wide variety of rules that define the feasibility of a route or a
schedule. Secondly, because it does not possess the integrality property (i.e., there may be a positive gap between its optimal value and that of its linear relaxation) as discussed in Desrosiers et al. (1984), the column generation approach can derive tighter bounds than those obtained from the linear relaxation of arc-based formulations. Thirdly, there exist efficient algorithms at least for some important variants of the SPPRC.

The SPPRC was introduced in the Ph.D dissertation of Desrochers (1986) as a subproblem of a bus driver scheduling problem. It consists of finding a shortest path among all paths that start from a source node, end at a sink node, and satisfy a set of constraints defined over a set of resources. A resource corresponds to a quantity, such as the time, the load picked-up by a vehicle, or the duration of a break in a work shift, that varies along a path according to functions, called resource extension functions (REFs). A REF is defined for every arc in the network and every resource considered. It provides a lower bound on the value that the corresponding resource can take at the head node of the corresponding arc, given the values taken by all the resources at its tail node. The resource constraints are given as intervals, called resource windows, which restrict the values that can be taken by the resources at every node along a path. Such a constraint is defined for every node in the network and every resource considered.

Figure 2.1 provides an SPPRC example that involves the resource time. The source and sink nodes are denoted by \( s \) and \( t \), respectively. Each arc \((i,j)\) bears a two-dimensional vector: The first component \( t_{ij} \)