

Chapter 1

ADVERTISING DIRECTED TOWARDS EXISTING AND NEW CUSTOMERS

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Abstract This paper considers a specific marketing problem based on a model by Gould (1970). The extension is that we have two kinds of advertising directed towards new customers and existing customers, respectively. We found that history dependent behavior occurs: if initial goodwill is small then it does not pay to spend a lot of money on advertising towards existing customers. Consequently convergence to a saddle point with low goodwill prevails where there is only advertising with the aim to attract new customers. On the other hand, for larger initial goodwill, eventually a steady state with a high goodwill level is reached where both types of advertising are used.

1. Introduction

Dynamic advertising models are among the first applications of Pontryagin's maximum principle in the economics and management area. The first comprehensive survey of the dynamic advertising literature was given by Sethi (1977a). It was devoted to determining optimal advertising expenditures over time subject to some dynamics that defines how advertising expenditures translate into sales and in turn into profits for

a firm or a group of firms under consideration. More than fifteen years later, this survey was updated by Feichtinger, Hartl and Sethi (1994).

The surveys by Sethi (1977a) and Feichtinger, Hartl and Sethi (1994) were organized in four and five model categories, respectively, the first two of which were *advertising capital models* and *sales-advertising response models*. Advertising capital models considered advertising as an investment in the stock of *goodwill* as in the model of Nerlove and Arrow (1962). Sales-advertising response models are characterized by a direct relation between the rate of change in sales and advertising and represent various generalizations of the descriptive model due to Vidale and Wolfe (1957).

Advertising capital models typically are extensions and/or modifications of the early seminal dynamic advertising model due to Nerlove and Arrow (1962). They consider a stock of *advertising goodwill*, which summarizes the effects of current and past advertising expenditures by a firm on the demand for its products. The advertising capital changes over time according to “investments” by current advertising and by a constant proportional depreciation rate. The objective of the monopolistic firm is to maximize the present value of net revenue stream discounted at a fixed interest rate. Since the price does not enter the system dynamics in these models, it can be determined by static maximization of the profit function so that the resulting optimal control model has only advertising as a single control variable. In case revenue is proportional to goodwill, the optimal advertising policy in this linear problem is characterized by a *most rapid approach* to a singular goodwill level; see e.g. Sethi (1977b), and Hartl and Feichtinger (1987). Several nonlinear and other extensions have been proposed. In the model by Gould (1970) revenue is a concave function of goodwill which leads to a smooth optimal advertising policy and an *asymptotic convergence* to an equilibrium advertising capital stock.

The second class of models are *sales-advertising response* models. These models are characterized by a direct relation between the rate of change in sales and advertising in the form of a differential equation. The basic advertising model by Vidale-Wolfe (1957) assumes that increases in sales are proportional to advertising expenditure, u , the captured fraction of the market potential, x , and the remaining market potential, $1 - x$. As in the goodwill models, a constant decay rate is assumed. The dynamics are fundamentally different from the advertising capital dynamics because of the presence of the terms $u(1 - x)$ and $ux(1 - x)$ in place of the term u . While Gould (1970) had analyzed the problems both with diffusion dynamics in the presence of convex advertising cost, his treatment was not exhaustive. More specifically, he obtained a single