Chapter 10

# NUMERICAL SOLUTIONS TO LUMP-SUM PENSION FUND PROBLEMS THAT CAN YIELD LEFT-SKEWED FUND RETURN DISTRIBUTIONS

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**Abstract** The paper is about pension fund problems where an agent pays an amount $x_0$ to the fund manager and is repaid, after time $T$, a lump sum $x(T)$. Such problems admit an analytical solution for specific, rather unrealistic formulations. Several practical pension fund problems are converted in the paper into Markov decision chains solvable through approximations. In particular, a couple of problems with a non-differentiable asymmetric (with respect to risk) utility function are solved, for which left-skewed fund-return distributions are reported. Such distributions ascribe more probability to higher payoffs than the right-skewed ones that are common among analytical solutions.

## 1. Introduction

This paper[1] is about lump-sum pension fund problems *i.e.*, such where an amount $x_0$ is paid to the fund manager by an agent who is repaid, after time $T$, a lump sum $x(T)$. The latter is called here the pension.

The purpose of this paper is to propose approximately optimal solutions to several such pension fund problems where performance measures are asymmetric with respect to risk. We will show that the fund return distributions obtained for those measures can be left skewed (negatively). Such distributions ascribe *more probability to higher payoffs* than the right-skewed ones that are common among analytical solutions to prob-

[1]This paper draws from Krawczyk (2003).



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lems characterised by the HARA (Hyperbolic Absolute Risk-Aversion) family of utility functions. If most utility realisations (mode) are low then a policy which leads to that result might not be acceptable for a realistic fund manager.

The problem of how to produce an acceptable portfolio-performance strategy is well recognised in the *static* context. Markowitz (1952) is credited with pioneering the classical mean-variance portfolio selection problem whose solution balances a good average yield with a certainty of achieving it. Since Markowitz (1959) seminal book, many authors have worked on extensions and other methods for portfolio optimisation that would allow for hedging against uncertainties and/or assure an acceptable level of payoff; see Bertsimas *et al.* (2004), de Athayde & Flôres Jr. (2004), Rockafellar & Uryasev (2000) or Krawczyk (1990).

On the other hand, the mainstream research onto *dynamic* portfolio management has been following the seminal works by Samuelson (1969) and Merton (1969) (also Merton (1971)) and concentrated on solutions to the HARA problems. Such solutions typically provide an optimal strategy in closed form. The solutions are generically risk-sensitive but leave aside the utility distribution issues; see *e.g.*, Brennan *et al.* (1997), Fleming & Sheu (2000), Morck *et al.* (1989).

Recently, problems of hedging and/or assurance of attainment of a desired payoff (thus "realistic") have been investigated in the context of dynamic portfolio management. For example, Howe *et al.* (1996) discuss multiperiod minimax hedging strategies, Gülpinal *et al.* (2004) apply mean-variance analysis to multistage portfolio management and Frey & Runggaldier (1999) apply risk-minimising hedging approach to dynamic strategy determination. Most importantly for "certainty" of payoff achievement, Value-at-Risk and Conditional Value-at-Risk constraints have been used in Yiu (2004) and Bogentoft *et al.* (2001), respectively, to produce acceptable management policies in the dynamic context.

Basak & Shapiro (2001) consider two types of "realistic" portfolio managers: "VaR-RM" — that incorporate a Value-at-Risk constraint in their optimisation problem formulation and "LEL-RM" — that limit their expected losses, and who appear very much like the Conditional Value-at-Risk managers from Bogentoft *et al.* (2001). Basak & Shapiro (2001) obtain closed form solutions for a CRRA utility function for either manager and compare performance of their portfolios. The problem with a Value-at-Risk constraint yields a solution in which while (as expected) the Value-at-Risk is limited, larger losses will be incurred "in the most adverse states of the world" than under no VaR constraint. The authors show that this is because a VaR risk manager minimises the