Chapter 16

A STOCHASTIC OPTIMAL CONTROL POLICY FOR A MANUFACTURING SYSTEM ON A FINITE TIME HORIZON

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Abstract We consider a problem of optimal production control of a single reliable machine. Demand is described as a discrete-time stochastic process. The objective is to minimize linear inventory/backlog costs over a finite time horizon. Using the necessary conditions of optimality, which are expressed in terms of co-state dynamics, we develop an optimal control policy. The policy is parameterized and its parameters are calculated from a computational procedure. Numerical examples show the convergence or divergence of the policy when the expected demand is greater or smaller than the production capacity. A non-stationary case is also presented.

1. Introduction

Uncertainty of demand is one of the major factors affecting decision-making in production planning and control (Nahmias, 2001). The problem of finding optimal control policies even for simple manufacturing systems under uncertainty has proved to be challenging both at the modeling stage and in analysis. In continuous time, hedging point policies have proved to be optimal for a class of stochastic systems. For such policies, a machine produces: a) at full capacity if the inventory level is

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lower than the hedging point; b) nothing if the inventory level is higher than the hedging point; and c) as much as the demand if the inventory level is equal to the hedging point.

The value of the optimum hedging point for systems with one unreliable machine and one part-type was first obtained by Akella and Kumar (1986), for the discounted cost problem and by Bielecki and Kumar (1988) for the average cost problem. These works assumed exponential distribution of the machine's work and repair time, constant deterministic demand rate, and linear surplus/backlog cost structure. Other researchers such as El-Ferik et al. (1998), Feng and Yan (2000), and Gershwin (1994) considered more general production environments and achieve only partial characterization of optimal policies. A comprehensive survey of research in optimal control of stochastic manufacturing systems can be found in Sethi et al. (2002). Perkins and Srikant (1997), extended the problem of Akella and Kumar to the case of two-part type and obtained an optimal policy. Perkins and Srikant (1998), subsequently enlarging upon their previous research into the problem of multiple part type, presented new results about the structure of the optimal policy and provided bounds on the optimal hedging points.

In this paper, we formulate a continuous-time optimal control policy when demand is discrete-time. The policy is not stationary, since the system is considered on a finite horizon. Within the intervals between demand realizations, the policy takes a specific form, different from hedging. It is proved to be optimal with the aid of the optimality conditions. We focus on developing a computation procedure for finding the control policy at each interval between demand realizations and on implementing the procedure for both the cases when demand distribution is stationary and with changes in time.

The paper is organized as follows: Section 16.2 introduces the system model and notation. An optimality condition is also provided at the end of the section. In Section 16.3 an optimal solution is characterized and parameterized. An algorithm for calculating the exact characteristics and parameters of the solution is presented. Section 16.4 provides numerical examples. Finally, we conclude the paper in Section 16.5.

2. Problem formulation and optimality conditions

Consider a machine whose production is intended to track an uncertain demand over a finite time horizon, $\tau \in [0, T]$. The machine is assumed reliable and no other source of uncertainty (except for demand realizations) is relevant. Let ω be a scenario of demand realizations,