Chapter 17

ON A STATE-CONSTRAINED CONTROL PROBLEM IN OPTIMAL PRODUCTION AND MAINTENANCE

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Abstract We consider a control problem introduced by Cho, Abad and Parlar (1993) which "incorporates a dynamic maintenance problem into a production control model". For a quadratic production cost function we present a detailed numerical study of optimal control policies for different final times. The maintenance control is either composed by bang-bang and singular arcs or is purely bang-bang. In the case of a linear production cost, we show that both production and maintenance control are purely bang-bang. A recently developed second order sufficiency test is applied to prove optimality of the computed controls. This test enables us to calculate sensitivity derivatives of switching times with respect to perturbation parameters in the system. Furthermore, numerical results are presented in the case where a state constraint on the number of good items is added to the control problem.

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1. Introduction

Cho, Abad and Parlar (1993) have considered a production process whose performance declines over time in the absence of maintenance. Preventive maintenance may be applied to the process to slow down the rate of decline of (or improve) the process performance which is measured in terms of good (non-defective) units of items produced. The authors restrict the analysis of optimal control solutions to a quadratic production cost function while the maintenance cost function is assumed to be linear. Using an algorithm that is tailored to the specific control problem, they compute a maintenance control which is composed of bang-bang and singular arcs while the production control is a continuous function.

The present paper pursues *three objectives* which are discussed in Sections 17.3-17.5. The *first objective* (Section 17.3) is to present numerical results that improve on those in Cho, Abad and Parlar (1993). We apply recently developed optimization techniques to obtain feasible controls that satisfy the necessary optimality conditions in Pontryagin's maximum principle with high accuracy. For a variety of finite time horizons, different control structures are elaborated. Surprisingly, no sufficient conditions are available in the literature which would bear upon the type of controls encountered in the model.

The *second objective* (Section 17.4) is the computation of optimal controls in the case where both the maintenance and production cost functions are linear. It is shown that the production and maintenance controls are purely bang-bang. In addition, we are able to verify numerically that the computed bang-bang controls provide a strict strong minimum. This is achieved by applying a new sufficiency test developed in Agrachev, Stefani and Zezza (2002), Maurer and Osmolovskii (2003,2004), Maurer, Büskens, Kim and Kaya (2004) and Osmolovskii and Maurer (2005).

The *third goal* is discussed in Section 17.5 where an additional state constraint, a lower bound for the number of good items, is imposed. We show that the optimal control solution contains one boundary arc and is bang-bang on interior arcs. To prove sufficient optimality conditions, we extend the second order test in Section 17.4 to handle a boundary arc.