Chapter 18

RELIABILITY INDEX

A. Bensoussan
University of Texas at Dallas, USA
Alain-Bensoussan@utdallas.edu

Abstract The reliability index is a useful indicator to compute the failure probability. If J is the performance of interest and if J is a Normal random variable, the failure probability is computed by

$$P_f = N(-\beta)$$

and β is the reliability index. When J is a nonlinear function of n normal random variables $(X_1, \ldots, X_n)$, then the preceding formula can be generalized, with some approximation. One uses a nice property of the reliability index, to be the shortest distance of the origin to the failure region. This method introduced by B.M. Ayyub, provides an analytic alternative to the Monte Carlo method.

1. Introduction

The objective of this article is to discuss a method to compute the failure probability (or its opposite the reliability) of an element, subject to several random inputs. It is an analytical approach, which can be compared to the Monte Carlo approach, common in this type of problem. This work relies on the presentation of B M. Ayyub (2003), where the method is introduced. We give a rigorous treatment of the main results.

2. Reliability Assessment

The reliability of an element of a system can be determined based on a performance function. Call J this performance function. It is assumed to be a function $J(X_1, \cdots, X_n)$, where the X_i are random variables. The limit state is when $J = 0$. When $J < 0$, the element is in a failure state, whereas when $J > 0$, it is in the survival state. The failure probability

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is defined as

$$P_f = P(J < 0)$$

Of course,we can write

$$P_f = \int \cdots \int \{J < 0\} f_{X_1,\cdots,X_n}(x_1, \cdots, x_n) dx_1 \cdots dx_n$$

where

$$f_{X_1,\cdots,X_n}(x_1, \cdots, x_n)$$

represents the joint probability density of the random variables. This formula is hardly computable, and one relies naturally of the Monte Carlo approach to estimate it. The reliability index method that we are going to develop is an alternative, which can be less costly than the Monte Carlo approach, especially when the variables are gaussian, or close to gaussian.

3. The case of a linear performance with two inputs

This case permits to understand the essence of the method. We assume that

$$J = S - L$$

If we are in the domain of materials, S represents the structural strength of the material, and L the load effect. If we are in the economic field, then S can represent the supply and L the demand. S, L are random variables. Let μ, σ denote the mean and standard deviation of J. One defines

$$\beta = \frac{\mu}{\sigma}$$

as the reliability index. If J is normally distributed and $N(x)$ represents the cumulative distribution of the standard normal variable, then one has

$$P_f = N(-\beta) = 1 - N(\beta)$$

If S, L are normally distributed, with means μ_S, μ_L, standard deviations, σ_S, σ_L and if they are not correlated, then one has the formula

$$\beta = \frac{\mu_S - \mu_L}{\sqrt{\sigma_S^2 + \sigma_L^2}}$$

We assume $\mu_S - \mu_L > 0$, so the reliability index is positive. Define the reduced variables

$$Y_S = \frac{S - \mu_S}{\sigma_S}, Y_L = \frac{L - \mu_L}{\sigma_L}$$