Chapter 3

CAPITAL RESOURCE SUBSTITUTION, OVERSHOOTING, AND SUSTAINABLE DEVELOPMENT

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Abstract We study an optimal control problem with a man-made capital stock, and a stock of renewable natural resource. They are substitutable inputs in the production of the final good. Starting from low levels of both stocks, the optimal policy consists of three phases. In phase I, the planner builds up the stock of resource above its steady state level, while the man-made capital stock is kept below its steady state level. In phase II, the resource stock declines steadily, while the man-made capital stock continues to grow, until the steady state is reached, and the economy stays thereafter. The model exhibits "overshooting" property.

1. Introduction

Since man-made capital and natural resources are substitutable inputs in the aggregate production function, a natural question that arises is how to optimally accumulate capital and manage the resource stock. The case where the natural resource stock is non-renewable has been studied by Solow under the the maximin criterion, and Dasgupta and Heal (1979) and Pezzy and Withagen

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(1998) under the utilitarian criterion. Solow assumed a Cobb-Douglas production function, and showed that if the share of capital is greater than the share of natural resource, then a constant path of consumption is feasible, and along such a path, the man-made capital stock increases without bound. Dasgupta and Heal (1979) and Pezzy and Withagen (1998) showed that, under the utilitarian criterion, the man-made capital stock will reach a peak, and afterwards both stocks fall to zero asymptotically.

In this paper, we study the optimal path for an economy that produces an output using a stock of capital and a resource input extracted from a stock of renewable natural resource. We retain the Solow-Dasgupta-Heal assumption that capital and resource are substitutable inputs in the production of the final good, but our model differs from theirs because the resource stock is renewable. We wish to find the optimal growth path of the economy under the utilitarian criterion. We show that there exists a unique steady state with positive consumption. We ask the following questions: (i) Can it be optimal to get to the steady state in finite time under the assumption that the utility function is strictly concave? (ii) Can finite-time approach paths to the steady state be smooth, in the sense that there are no jumps in the control variables? (iii) Are there non-smooth paths to the steady state?

The answers to the above questions are as follows.

There exists a set of initial conditions (which forms a one-dimensional manifold, i.e., a curve, in the state space) such that the approach path to the steady state takes a finite time, and is smooth. The path along the manifold toward the steady state involves a steady accumulation of the capital stock, and a steady running down of the resource stock toward its steady state level.

If the initial conditions are not on that one-dimensional manifold, then it may be optimal to get to some point on that manifold first, and then move along the manifold to get to the steady state. The path that gets to a point on the manifold is not smooth at the time it meets the manifold.

We show that starting from low levels of capital stock and resource stock, the optimal policy consists of three phases. In phase I, the planner builds up the stock of resource above its steady state level, while the capital stock is kept below its steady state level. In phase II, the resource stock declines steadily, while the capital stock continues to grow, until the steady state is reached. In phase III, the economy stays at the steady state. Thus, our model exhibits the "overshooting" property.

Before proceeding, we would like to note that there are a number of articles that are somewhat related to our paper, where the authors discussed thr optimal use patterns for renewable resources and the sustainability of economies. Clark et.al. (1979) provided a general formulation with irreversible investment. They focussed on irreversibility, and did not obtain an "overshooting" result. Among the relatively recent papers, Bertratti et.al.(1998) addressed the prob-