Chapter 4

HIERARCHICAL AND ASYMPTOTIC OPTIMAL CONTROL MODELS FOR ECONOMIC SUSTAINABLE DEVELOPMENT *

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Abstract In this brief paper one shows the relevance of asymptotic control theory to the study of economic sustainable development. One also proposes a modeling framework where sustainable economic development is represented through a paradigm of optimal stochastic control with two time-scales. This shows that several contributions of rof. Sethi, in the domain of hierarchical and multi-level control models in manufacturing and resource management can also serve to better understand the stakes of sustainability in economic growth and to assess long term environmental policies.

1. Introduction

The theory of economic growth has been using a lot of control theory (and calculus of variations) to attain in the late seventies a remarkable scientific status in the domain of economics and social sciences. In particular the theory of Hamiltonian systems has been used to explain the asymptotic behavior of a growing economy, the key feature being that the economic growth models were likely to exhibit global attractors, both for the state and costate variables. This attractor was called "turnpike", following the work by Cass (1995), McKenzie (1976), Brock & Scheinkman (1976), Rockafellar (1973) and many others. An optimally growing economy in a stationary environment is prone to converge to an

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extremal[1] steady-state which would then characterize a sustainable optimal economic state. The turnpike property has also been an important feature in the study of existence, sufficiency conditions and asymptotic behavior of optimal control problems with zero-discount and overtaking optimality criterion. (see Brock and Haurie (1976) and Carlson, Haurie and Leizarowitz (1994)). In stochastic decision models the turnpike property has been associated with the existence of stationary or invariant state probability measures that are defined by optimal policies (see e.g. Puterman (1994), Veinott (1964)).

An interesting feature of this theory was the result showing that when the pure time preference discount rate $\rho > 0$ was increasing, the sufficient conditions for observing the turnpike property were more stringent. In brief, high discounting could jeopardize sustainability. The economics of the environment and, more precisely, cost-benefit analysis for global climate change mitigation has generated a renewed interest for low discounting or even zero discounting in economic growth models. For example, in Weitzman (1998) Weitzman explained why one should use the lowest rate to discount distant futures. In an interesting monograph edited by Portney and Weyant (1999) several leading authors discuss the proper discount rate to use when dealing with environmental problems that will affect several generations down in the future.

In this note one intends to show that the turnpike theory has an important potential to contribute to a better understanding of the long term economic growth under a global environmental threat and, more generally, of the elusive concept of economic sustainability. Considering the purpose of this essay, most technical developments have deliberately been avoided and the interested reader is referred to the sources given in reference. The paper is organized as follows: Section 4.2 shows the relevance of turnpike theory for sustainable development analysis. One illustrates this property by showing how it would appear in the most popular cost-benefit-analysis (CBA) models, in particular the Nordhaus and Nordhaus-Boyer models DICE-94 or DICE-99 (Nordhaus (1992), Nordhaus (1994), Nordhaus and Boyer (2000)). Section 4.3 shows how the climate change and climate damage uncertainty can be introduced in this class of models, using a two time-scale, hierarchical control scheme. A *limit climate control problem* is formulated in the slow time-scale which serves to determine the optimal long term GHG emissions cap. An auxiliary optimal economic growth problem is then formulated, in the fast time-scale with incentives based on the potential function associated with the solution of the limit climate control problem. The solution of this *transient optimal economic growth* problem will drive the economy toward the long term sustainability goal.