Chapter 5

COMMON PROPERTY RESOURCE AND PRIVATE CAPITAL ACCUMULATION WITH RANDOM JUMP

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Abstract We present a model of exploitation of a common property resource when agents can also invest in private and productive capital. The resource extracted from a common pool is non-renewable, but the resource stock is under uncertainty in the sense that the stock might follow jump process. We show that there exists an optimal solution in the model.

1. Introduction

In Long and Katayama (2002) they presented a model of exploitation of a common property resource, when agents can also invest in private and productive capital. The resource extracted from a common pool is non-renewable in the model. We try to extend their result to the case where a common pool is under uncertainty in the sense that it could have a sudden increase or decrease in the process of extraction.

C. Deissenberg and R.F. Hartl (eds.), Optimal Control and Dynamic Games, 77–84.

The extension is quite natural when we see the present state of international crude oil market. Some producing countries encountered the technological difficulties of extraction and/or social hazards. Also the past history showed the unexpected discovery of new oil reserves. Yet the total reserve in the earth planet is limited, and it is expected that the resource is finally exhausted. However, people can accumulate man-made capital for substituting the exhaustible resource and extend the period in which the resource is utilized before it is completely depleted.

Considering these intrinsic aspects of resource economy, we present a model of uncertainty in the process of extraction of the resource and build the capital to substitute for the exhaustible resource. To incorporate it we build a model with a random jump in the stock of the resource.

The main issue is to see whether there is an optimal solution to this model.

2. The model

There are n identical agents having common access to a stock of non-renewable natural resource, denoted by $S(t)$. Each agent i also owns a private capital stock $K_i(t)$. Agent i extracts the amount $R_i(t)$ of the common resource stock ($i = 1, ..., n$). Extraction is costless. Total extraction in the economy at time t is $R(t) = \sum_{i=1}^{n} R_i(t)$, and the reserve depletes according to

$$\dot{S}(t) = -R(t)$$

if it is not subject to any uncertainty.

First assume that each individual extracts equal amount, and so it follows that

$$\dot{S}(t) = -nR_i$$

However, the reserve may be augmented or damaged several times in the finite horizon and the reserve size is affected by those jumps in magnitude.

The jump process takes the form $dJ(t)$, and the resource stock is governed by

$$dS(t) = dJ(t) - nR_i dt \tag{5.1}$$

The stock level at time t is

$$S(t) = S_0 + J(t) - \int_0^t nR_i(s)ds$$

where $J(t)$ is a pure jump process given by

$$J(t) \triangleq \int_0^t \int_{R\setminus\{0\}} S(s-) \cdot zN(ds, dz)$$