Chapter 7

CHARACTERIZING DYNAMIC IRRIGATION POLICIES VIA GREEN'S THEOREM

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Abstract We derive irrigation management schemes accounting for the dynamic response of biomass yield to salinity and soil moisture as well as for the cost of irrigation water. The simple turnpike structure of the optimal policy is characterized using Green's Theorem. The analysis applies to systems of arbitrary end conditions. A numerical application of the turnpike solution to sunflower growth under arid conditions reveals that by selecting the proper mix of fresh and saline water for irrigation, significant savings on the use of freshwater can be achieved with negligible loss of income.

1. Introduction

Increasing water scarcity and the alarming deterioration in the quality of many freshwater resources call for improved irrigation efficiency to sustain viable agriculture over vast areas around the globe. Using water of lesser quality and continuously adjusting irrigation rates to the varying needs of the growing

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plants can save a significant fraction of freshwater used in traditional irrigation practices. The trade-offs between the cost of water and the essential contribution of suitable soil moisture to biomass growth give rise to optimization problems that are both theoretically interesting and practically relevant.

An Optimal Control analysis of a dynamic irrigation problem accounting for soil moisture and biomass growth dynamics as well as for the associated cost of irrigation water is presented in Shani, Tsur and Zemel (2004), where the ensuing optimal policy is shown to take a particularly simple form. The policy is defined in terms of two parameters: a turnpike soil moisture $\hat{\theta}$ and a stopping date, such that the optimal moisture process, $\theta(t)$, must be brought from its initial level to the turnpike $\hat{\theta}$ *as rapidly as possible* and maintained at that level until the stopping date, at which time irrigation ceases and the plants are left to grow on the remaining soil moisture until the time of harvest. This simple turnpike behavior is neither unique to the irrigation problem nor is it rare in the dynamic optimization literature. Similar characterizations have been derived for a large variety of economic and management problems (see e.g. Vidale and Wolfe (1957), Sethi (1974), Haruvy, Prasad and Sethi (2003)) and explained by geometrical considerations using Green's Theorem (see Miele (1962), Sethi (1973), Sethi (1977), Sethi and Thompson (2000)). In this method, one eliminates the control, replaces the line integral in the objective functional by an area integral, and compares the values obtained from any two feasible policies by analyzing the sign of the integrand over the area encircled by the trajectories corresponding to these policies. The similar characteristics of the optimal irrigation policy suggest that this problem can also be analyzed in terms of Green's theorem. It turns out that certain features of the irrigation problem render the application of Green's Theorem non trivial in this case, and novel considerations must extend the method to derive the optimal policy.

The purpose of the present work is twofold. First, we adapt the standard Green's Theorem analysis to more complex situations of arbitrary end conditions. Applying the method to the irrigation problem, we explain the simple structure of the optimal policy and provide new links relating the method and Optimal Control theory. Second, we extend the original model of Shani, Tsur and Zemel (2004) by incorporating salinity effects. In many arid regions, brackish water is available to replace scarce freshwater resources for irrigation purposes. The growers, however, must also account for the reduction in yield implied by increased salinity in the root zone. We find that by carefully adjusting the salinity of the irrigation water mix and the parameters of the turnpike policy, the growers can increase the net income from their crop and, more importantly, mitigate freshwater scarcity (manifest in terms of exogenous freshwater quota imposed for each growing season) with only minor income loss.