CHAPTER 27

FULL COMMITMENT CONTRACTS WITH INTERDEPENDENT PERIODS

This is the third of four chapters that examine multi-period principal-agent models. As in Chapters 25 and 26, we assume the principal and the agent can commit to a long-term contract without subsequent renegotiation. In this chapter, as in Chapter 26, we relax the Chapter 25 assumptions that the performance reports are stochastically and technologically independent. The key innovations pertain to the exploration of the impact of transforming performance measures to achieve stochastic independence, characterization of optimal non-linear contracts, creation of indirect covariance incentives by allowing the second-period incentive rates to vary with the first-period performance reports, the use of effort cost risk insurance and risk-premium risk insurance, and the consideration of productivity information.

We begin in Section 27.1 by examining some basic issues in sequential choice. To explore these issues, in Section 27.1.1 we formulate a two-period model that is a special case of the basic model introduced in Section 25.1. This model is less general than the basic model, but it is sufficiently general to encompass both stochastic and technological interdependence. A key point in this section is that one must be careful in specifying the incentive compatibility constraints when the agent makes sequential choices. Of particular concern is the potential for "double shirking," which refers to the agent's strategy in the second period if he deviates from the planned action in the first period. The deviation takes him "off the equilibrium path," and, to be a sequential equilibrium, the incentive constraints must be such that they reflect his rational response if he finds himself on that path.

Section 27.1.2 briefly describes three special cases in which there is stochastic interdependence, so that the first-period reports are informative about both the first-period action and about future random events. The three types of random events are: additive noise, payoff productivity, and performance productivity.

Chapter 26 examines the correlated additive noise case within a *LEN* model. Section 27.2 introduces transformations of the normally distributed performance measures such that the revised representations continue to be normally distributed, but are stochastically independent. The revised measures are referred to as *stochastically independent sufficient performance statistics*. In Section 27.2.1, the transformation merely orthogonalizes the noise terms, whereas in

Section 27.2.2 the transformation normalizes the statistics so that they have zero means. While creating stochastically independent statistics can simplify the analysis, the transformation generally creates technological interdependence. As illustrated using a simple two-period *LEN* model in Section 27.2.1, orthogonalizing two technologically independent, stochastically correlated measures produces two stochastically independent but technologically interdependent performance statistics. If the linear contract is expressed in terms of the original measures, then the induced first-period action depends entirely on the first-period incentive rate. However, with the statistics, the induced first-period action depends directly on the first-period incentive rate and indirectly on the second-period incentive rate.

Section 27.2.1 examines two examples. The first is an auto-regressive process that is technologically and stochastically interdependent. It is noteworthy that, in this case, orthogonalization provides statistics that are both stochastically and technologically independent. The second example is a stock price process, for which the orthogonalized statistics are excess returns. These returns are stochastically independent, but they are not likely to be technologically independent.

Orthogonalized statistics work well in the *LEN* model in which the actions do not vary with the information received. However, if the actions vary with the information received, it is useful to normalize as well as to orthogonalize the performance measures. The normalization process described in Section 27.2.2 requires the use of the principal's conjectures with respect to the agent's actions, including the principal's conjecture with respect to how the agent's actions will vary with the information received, given the contract between the principal and the agent.

In the *LEN* model, the optimally induced actions are independent of prior information – they are constants. This is, in part, a result of the fact the *LEN* contract is constrained to be linear. Section 27.3 considers a model in which the preferences and performance measures are the same as in the *LEN* model, but the contract need not be linear. Section 27.3.1 explores the nature of the optimal contract (when the form of the contract is not constrained). Key features of the optimal contract include second-period incentives that vary with the first-period performance report, effort-cost risk insurance, and an additional indirect first-period covariance incentive not present in the *LEN* model.

The characterization of the optimal contract is complex, and does not lend itself to comparative statics. Section 27.3.2 considers a more tractable contract that permits inducement of actions that vary with the information received. The linearity constraint of the *LEN* contract is relaxed by allowing the second-period incentive rate to be a linear function of the first-period performance statistic. In addition, the second-period "fixed" wage can vary with the first-period performance statistic so as to compensate the agent for his second-period effort cost and risk premium, conditional on the first-period report. This approach pro-