CHAPTER 19

LINEAR CONTRACTS

A compensation contract is defined to be linear if there exists a constant f and another constant or vector v such that $\boldsymbol{c}(y) = f + v \cdot y$, whenever $f + v \cdot y \in C = [\underline{c}, \infty)$ and $\boldsymbol{c}(y) = \underline{c}$ otherwise, where y is perhaps a multi-dimensional performance measure and $\underline{c}$ is an exogenously imposed lower bound on consumption. The use of linear contracts in agency theory is appealing on two grounds. First, restricting our analysis to linear contracts makes some analyses more tractable and some results more intuitive. Second, many contracts observed in the "real world" appear to be linear or at least piecewise linear.

There are two basic approaches to the use of linear contracts in the agency theory literature. The first approach is to restrict the analysis to the set of linear contracts, whether the optimal contract is linear or not. In the early years of agency theory this would have been viewed as a major flaw in any research paper. However, as we have learned more about the implications of optimal contracts, the perspective has shifted such that it is now the view of many researchers (including ourselves) that a simplification to the set of linear contracts is justified, if the analysis provides insights that are believed not to be confined to settings with linear contracts. In Section 19.1 we review this approach in the simple setting in which both the performance measure and the agent's action are single-dimensional. In Chapter 20 we consider settings in which the performance measure and the agent's action are both multi-dimensional. Chapter 21 reviews models in which one of the performance measures is a market based performance measure such as the stock price. The second approach is to restrict the analysis to conditions under which the optimal contract is linear. However, as we shall see in Section 19.2, this only occurs under highly specialized conditions on preferences, beliefs, technology, and information.

Sufficient Conditions for the Optimal Contract to Be Linear

Before we proceed into the main analysis of settings with linear contracts, it is useful to review sufficient conditions for linear compensation contracts to be optimal within the basic principal-agent model. Of course, if the performance measure is binary, any compensation function can be expressed as a linear function of the performance measure. Hence, we generally assume that the performance measure has more than two possible signals. In the analysis that follows we further assume that the principal owns the outcome and is risk neutral, the

agent's utility function is $u^a(c,a) = u(c)k(a) - v(a)$, with $u'(c) > 0$, $u''(c) < 0$, $k(a) > 0$, $k'(a) < 0$, $v'(a) > 0$, and either $k(a) = 1$ or $v(a) = 0$, for all a. The agent's consumption set is $C = [\underline{c}, \infty)$, the set of actions A is a convex set on the real line, i.e., $A = [\underline{a}, \bar{a}]$, and we assume the agent's incentive constraint can be represented by its first-order condition. Given these assumptions, and considering $a \in (\underline{a}, \bar{a})$, the optimal contract is characterized by (see Chapter 17)

$$M(\mathbf{c}(y)) = \lambda k(a) + \mu[k(a)L(y|a) + k'(a)],$$

where
$$L(y|a) = \frac{\varphi_a(y|a)}{\varphi(y|a)}, \tag{19.1}$$

whenever the right-hand side is such that $M(\mathbf{c}(y)) \geq M(\underline{c})$.

Proposition 19.1

Sufficient conditions for the compensation contract to be linear are that $u(c) = \ln(\alpha c + \beta)$, $\alpha c + \beta > 0$, $\alpha > 0$, and $L(\cdot|a)$ is a linear function of y (e.g., $\varphi(y|a)$ is from the one-parameter exponential family).

The proof is straightforward, given (19.1) and the fact that with log-utility $u'(c) = \alpha/(\alpha c + \beta)$, which implies that $M(c) = c + \beta/\alpha$. However, with log-utility we must be careful if the signal space Y is convex in which case the first-best solution might be approximated arbitrarily closely if, for example, the performance measure is normally distributed (see Proposition 17.10).

Even though optimal contracts are linear with log-utility and linear likelihood ratios, this provides no significant advantage in terms of analytical tractability since there is no simple representation of the agent's expected utility. As we will see in the following section, linear contracts combined with exponential utility, on the other hand, is the "magic" combination for providing analytical tractability.

19.1 LINEAR SIMPLIFICATIONS

In this section we consider a setting sometimes referred to as the *LEN* framework which stands for "Linear contracts", "Exponential utility", and "Normally distributed performance measure."[1] That is, compensation contracts are exogenously restricted to the class of linear contracts, the agent's preferences are represented as a multiplicatively separable exponential utility function in c and

[1] The following analysis is similar to that found in Hughes (1988).