CHAPTER 20

MULTIPLE TASKS AND MULTIPLE PERFORMANCE MEASURES

In the preceding chapters we assumed that the set of actions A from which the agent must choose is either a finite or continuous set that is ordered such that more "effort" increases the "benefit" to the principal and increases the "cost" to the agent. We now recognize that an agent often performs multiple tasks and, therefore, his effort is more realistically described as a vector of actions $\mathbf{a} = (a_1, ..., a_m)^t$ such that a_j refers to the "effort" expended in task j. In this setting, there can be two actions $\mathbf{a}'$ and $\mathbf{a}''$ that have the same "cost" to the agent but different "benefits" to the principal. This shifts the focus from being only concerned about the optimal "intensity" of effort, to being concerned about both the "intensity" and "allocation" of the effort expended by the agent. When "intensity" is the only concern, the key criterion in selecting performance measures is the extent to which they help minimize the risk imposed on the agent to induce the desired intensity. However, when "allocation" of effort is also of concern, the choice of performance measures must consider both the "allocation" induced and the risk that must be imposed to induce a particular level of "intensity" and "allocation."

A performance measure that aligns the agent's effort allocation with the benefits to the principal is referred to as being *perfectly congruent* with the principal's objectives. A single performance measure's lack of perfect congruency or inclusion of noise can result in other performance measures having value because they complement the first measure either by improving the allocation of effort or by reducing the risk imposed to induce a particular allocation.

In Section 20.1 we introduce the elements of the basic multi-task model, which are similar to the formulation by Gjesdal (1982). Section 20.2 explores multi-dimensional versions of the *LEN* model introduced in Chapter 19. This simplified model provides a number of insights into the role of multiple performance measures in motivating multi-dimensional effort and identifies the factors that affect the relative weights placed on the performance measures. Finally, in Section 20.3, we explore some simple settings introduced by Holmström and Milgrom (1991) (HM), in which the form of the agent's cost function significantly influences the nature of the optimal incentives and effort allocation.

20.1 BASIC MULTI-TASK MODEL

We assume that the principal is risk neutral and "owns" the outcome x, while a risk and effort averse agent implements an m-dimensional action $\mathbf{a} \in A \subseteq \mathbb{R}^m$. The agent's preferences are represented by a separable utility function $u^a(c,\mathbf{a}) = u(c)k(\mathbf{a}) - v(\mathbf{a})$, which is defined over the agent's compensation $c \in C$ and his action $\mathbf{a}$. As before, the agent's action $\mathbf{a}$ is assumed to be non-observable by the principal. The contractible performance measures generated by system η consist of a vector of n performance measures, denoted , $\mathbf{y} = (y_1,...,y_n)^t \in Y \subseteq \mathbb{R}^n$, which may or may not include the outcome x. The compensation contract offered to the agent by the principal is $\boldsymbol{c}$: $Y \rightarrow C$. The expected outcome (benefit) to the principal given action $\mathbf{a}$ is given by $b(\mathbf{a}) \equiv \mathrm{E}[x|\mathbf{a}]$ and the homogeneous signal beliefs are represented by the probability distribution function $\Phi(\mathbf{y}|\mathbf{a},\eta)$.

20.1.1 General Formulation of the Principal's Problem

The principal's decision problem is formulated as follows.

$$U^p(\eta) = \underset{\boldsymbol{c},\mathbf{a}}{\text{maximize}} \quad U^p(\boldsymbol{c},\mathbf{a},\eta) \equiv b(\mathbf{a}) - \int_Y \boldsymbol{c}(\mathbf{y})\, d\Phi(\mathbf{y}|\mathbf{a},\eta), \tag{20.1}$$

$$\text{subject to} \quad U^a(\boldsymbol{c},\mathbf{a},\eta) \equiv \int_Y u^a(\boldsymbol{c}(y),\mathbf{a})\, d\Phi(\mathbf{y}|\mathbf{a},\eta) \geq \overline{U}, \tag{20.2}$$

$$\mathbf{a} \in \underset{\mathbf{a}' \in A}{\text{argmax}}\ U^a(\boldsymbol{c},\mathbf{a}',\eta), \tag{20.3}$$

$$\boldsymbol{c}(\mathbf{y}) \in C, \quad \forall\, \mathbf{y} \in Y. \tag{20.4}$$

Nothing of substance has changed from the general formulation in Chapter 18, except that we have replaced a with $\mathbf{a}$ and y with $\mathbf{y}$, which merely emphasizes that they are vectors. The significance of the multi-dimensional effort becomes more obvious if we assume that the set of possible actions is a convex set of the form $A = [\underline{a}_1,\overline{a}_1] \times ... \times [\underline{a}_m,\overline{a}_m]$. In that case, if the first-order conditions for the agent's decision problem characterize his action choice, then incentive constraint (20.3) is replaced by an $m \times 1$ vector of incentive constraints of the form,

$$\nabla_{\mathbf{a}} U^a(\boldsymbol{c},\mathbf{a},\eta) = \mathbf{0}, \tag{20.3'}$$