Chapter 1

ESSAY 1: EFFICIENCY INDICATORS AND INDEXES

This essay focuses on two approaches to measuring efficiency, namely the difference approach and the ratio approach. In the index number theory literature, Diewert (1998) classifies measures that are in difference form as **indicators** and measures that take the form of ratios as **indexes**, a terminology which we shall adopt here. As Diewert points out, the ratio approach is the 'traditional (bilateral) approach to index number theory.' Examples include the cost of living index (ratios of cost functions) as well as the familiar Paasche and Laspeyres price and quantity indexes. Diewert also points out that the difference approach pioneered by Bennet (1920) and Montgomery (1929, 1937) was largely forgotten, except for the difference quantity indicator, most familiar as a measure of consumer surplus.

A unifying feature of our approach to efficiency measurement is that whether difference or ratio based, they are all rooted in duality theory, which is also the basis by which we decompose our efficiency measures. The 'value' or dual measures are support functions such as profit, cost and revenue functions. Primal measures are their dual distance functions. This approach to efficiency measurement yields a natural correspondence between quantity and value measures. As we shall see below, the profit function with its additive structure finds its perfect match with the directional distance function which shares that structure. The more familiar cost and revenue-based Farrell efficiency indexes are multiplicative as are the Shephard type distance functions which are their duals. Eventually we shall see that the revenue and cost indexes and their duals are in fact special cases of the profit and directional distance

function, providing an elegant overarching structure.

This essay opens with the 'forgotten' indicator approach; we begin with a profit indicator and its decomposition. This section introduces the key technical efficiency component used in the indicator approach, namely the **Directional Distance Function**. This is followed with the parallel indicators for revenue and cost which also use special cases of the directional distance function. Next we turn to the ratio forms of efficiency indexes, including revenue and cost efficiency and their decompositions. The next section 'From Indicators to Indexes' links the two approaches; the final section shows how the ratio approach can be related to profits by employing the special case of hyperbolic efficiency and the modified profit concept of return to the dollar.

1. The Nerlovian Profit Indicator

Perhaps the most natural measure of performance that is based on differences is profit; so it follows that the natural form for a measure of **profit efficiency** is as a difference rather than as a ratio. This is also practical, since firms may earn zero profit, which poses problems in a ratio context. Thus we begin this essay by developing an indicator of profit efficiency which we dub the Nerlovian profit indicator along with technical and allocative component indicators.

As noted above, construction of a measure of profit efficiency based on ratios is impractical due to the fact that both maximal and observed profit may equal zero. The ratio of maximal to observed profit may be infinite, which is not meaningful. To avoid these problems, the Nerlovian profit indicator is defined as the difference between price deflated maximal profit and price deflated observed profit. The additive structure of the profit indicator carries over to its components; i.e., their sum equals the profit indicator.

We begin with some notation. Input quantities are denoted by

$$x = (x_1, \ldots, x_N) \in \Re_+^N$$

and their associated prices by

$$w = (w_1, \ldots, w_N) \in \Re_+^N.$$

The circumstance that inputs and their prices are real numbers implies that they are fully divisible. Inputs may be applied in any fraction and