

Chapter 3

ESSAY 3: AGGREGATION ISSUES

In this essay we study various issues related to aggregation of efficiency and productivity measures. The reader is referred to Blackorby, Primont and Russell (1978) for a general theoretical treatment of aggregation and separability in the context of modern microeconomic theory. The general idea is to find conditions on the fundamentals of technology or preferences which allow us to infer aggregate outcomes or performance measures (in our case) from individual measures of performance, for example.

Of course, the individual performance measures we have been discussing are themselves a type of aggregator function, summarizing or aggregating many attributes into a single number, often in the absence of prices, which for economists are the most natural tools to use in aggregating ‘apples and oranges.’ Instead of aggregating the many aspects of production of a single decision-making unit or producer into a single performance measure, here we begin by focusing on aggregating the individual performance measures up to a group or industry performance measure. For example, we ask: When can we simply ‘add up’ the individual measures—or take an average—to get a consistent measure of overall industry performance? As it turns out, aggregation of firm performance which is consistent with industry performance will require that we pay attention to the ‘functional form’ of the performance measures we are using. This general idea is illustrated with our opening motivating section on the Fox paradox.

The next sections go back to the indicators and indexes from Essay 1, and specifically derive conditions for consistent aggregation for

each case. The overall performance measures—for example the Nerlovian Profit indicator—are typically price dependent and lend themselves to adaptations of the aggregation result due to Koopmans (1957) concerning profit functions. The component measures, especially the technical efficiency components are less straightforward. We also include a discussion of aggregation using the Johansen approach and for Luenberger productivity indicators.

The next theoretical section addresses a different type of aggregation; namely, consistency of efficiency measures when we wish to aggregate across inputs or outputs. This is followed by a section devoted to maintaining consistency of the various decompositions when we wish to aggregate or average. We end the essay with an empirical application of profit efficiency and aggregation by Fukuyama and Weber.

1. The Fox Paradox

The purpose of this section is to show that the way efficiency indicators or indexes are defined and the way that the efficiency scores are aggregated are interdependent. A useful illustration arises from the Fox Paradox raised by K. Fox (1999). The essence of the paradox is that even if one firm produces each of two outputs more efficiently than another firm, when outputs are aggregated the first firm may be overall less efficient than the second.

To illustrate the paradox and how one may ‘outfox’ it, suppose that there are two firms $k = 1, 2$ each producing two outputs $m = 1, 2$. Let c_{km} be the observed or actual cost of producing output m by firm k , and let \hat{c}_{km} be the predicted or minimum cost. The efficiency of firm k in producing output m equals the ratio

$$\hat{c}_{km}/c_{km}, k = 1, 2 \ m = 1, 2. \quad (3.1)$$

Total cost for each firm is the sum of individual output costs, i.e.,

$$c_{k1} + c_{k2} \text{ and } \hat{c}_{k1} + \hat{c}_{k2}, k = 1, 2, \quad (3.2)$$

are the total observed and minimum cost, respectively for each firm.

Overall efficiency for each firm is defined as

$$(\hat{c}_{k1} + \hat{c}_{k2})/(c_{k1} + c_{k2}), k = 1, 2. \quad (3.3)$$

The Fox Paradox states that is possible that