Chapter 4

APPENDIX: AXIOMS OF PRODUCTION

The intention of this appendix is to make our book self-contained. Since most readers of these essays will have some familiarity with axiomatic production theory, this appendix is meant to serve as a reference or reminder of the relevant concepts. Consequently, we give only a rudimentary description of the basic axioms of production; for a more complete discussion see Färe and Grosskopf (1996), Färe and Primont (1995) or Shephard (1970).

We begin with the static production technology which transforms **Input Vectors**

$$x = (x_1, \ldots, x_N) \in \Re_+^N$$

into **Output Vectors**

$$y = (y_1, \ldots, y_M) \in \Re_+^M$$

and which can be represented in three equivalent ways: 1) by its graph, 2) by its output correspondence or 3) by its input correspondence.

The graph of **Technology** consists of all feasible input-output vectors, i.e.,

$$T = \{(x, y) : x \text{ can produce } y\}.$$

Thus if $(x, y) \in T$ then and only then can the input vector x produce the output vector y. For the single input-output case, the boundary of the technology is also known as the total product curve.

The **Output Correspondence** is the second representation of the technology—it is defined in terms of the graph as

$$P: \Re_+^N \to P(x) = \{y : (x, y) \in T\}. \tag{4.1}$$

The images of this correspondence are the **Output Sets** $P(x)$, which consist of all output vectors that can be produced by a given input vector $x \in \Re_+^N$. The definition above shows how these sets–also known as production possibility sets–and the output correspondence are defined in terms of the graph of technology, T.

Next we show that T can be retrieved from the output sets, thus proving that T and the output sets–or equivalently the output correspondence–provide the same information about the technology. Specifically, we can define T as

$$T = \{(x, y) : y \in P(x), x \in \Re_+^N\}.$$

The third representation of the technology is the **Input Correspondence** which may also be defined in terms of the graph

$$L: \Re_+^M \to L(y) = \{x : (x, y) \in T\}. \tag{4.2}$$

The images of the input correspondence $L(y), y \in \Re_+^M$ are called the **Input Requirement Sets** or **Input Sets** for short. An input set $L(y)$ consists of all input vectors that can produce the output vector y. The technology T can also be retrieved from the input sets, specifically

$$T = \{(x, y) : x \in L(y), y \in \Re_+^M\}. \tag{4.3}$$

The results above demonstrate the following proposition

Proposition A.1: $y \in P(x) \Leftrightarrow (x, y) \in T \Leftrightarrow x \in L(y)$,

which tells us that the output and input sets are equivalent representations of technology, as is T.

Proposition (A.1) is illustrated in Figure 4.1. The technology T consists of the input-output combinations (x, y) that are on and between the broken line and the x-axis. The input set associated with the output level y^o is $L(y^o)$ where $L(y^o) = [x^o, +\infty)$. The output set associated with input x^o is $P(x^o)$ where $P(x^o) = [0, y^o]$.

Under the axiomatic approach to production theory—which we follow here—the technology or production model is assumed to satisfy certain