Rotors can be provided with sensors and actuators that, through a suitable control system, can perform different tasks, from relieving the loads from the bearings to performing an active control of the dynamic behavior of the system. The control system, with its sensors and actuators, can act on only a few of the degrees of freedom of the rotor, for example to supply an additional damping, or it can fully control its position in space, as it occurs in a fully active magnetic suspension.

Magnetic bearings are perhaps the most common example of active control applied to the dynamics of rotors, but there are also many other applications, like controlled hydrodynamic supports, electrostatic bearings used in micromachines, or controlled pneumostatic bearings. In the following sections, only a brief introduction to the dynamics of controlled rotors will be presented, with particular reference to magnetic bearings, but the relevant equations can be applied to other devices.

A simplified linearized study of the behavior of rotors on magnetic bearings was presented in Section 8.5. There each magnetic bearing was considered as a linear spring-damper system, neglecting the dynamics of the sensors, actuators, amplifiers, and controller and neglecting the fact that in most applications, the sensors and the actuators are not collocated; i.e., the force exerted by the latter is applied in a point that does not coincide with the point where the sensor reads the displacement. These assumptions will be dropped in the present section.
16. Dynamics of controlled rotors

16.1 Open-loop equations of motion

16.1.1 Real coordinates

The general equations of motion of a rotating system that includes a number of actuators can be written in the form

$$\mathbf{M}\ddot{\mathbf{x}} + (\mathbf{C}_q + \mathbf{C}_u + \mathbf{G})\dot{\mathbf{x}} + (\mathbf{K} + \mathbf{K}\Omega^2 + \Omega\mathbf{C}_r)\mathbf{x} = \mathbf{f}_f + \mathbf{f}_e,$$  \hspace{1cm} (16.1)

where $\mathbf{f}_e$ and $\mathbf{f}_n$ are forcing vector functions of time due, respectively, to the actuators and to other causes. The latter includes also unbalance forces, which are harmonic in time with an amplitude proportional to $\Omega^2$ and a frequency equal to $\Omega$. Such an equation may contain the lateral behavior as well as the axial behavior in the case of a beam-like model, or may be much more complex in the case of 2D or 3D modeling. At any rate, at least the rotor must possess axial (or cyclic) symmetry because the equation has been written with reference to the fixed frame and has constant coefficients.

If the rotor is supported only by the actuators, like in the case of a rotor on active magnetic bearings, the stiffness matrix $\mathbf{K}$ is singular with four (six, if also the axial and the torsional behavior is included) vanishing eigenvalues because the four (six) rigid-body motions of the rotor are unconstrained. Also the rotating damping matrix $\mathbf{C}_u$ is singular. If it was not for the control forces, the rotor would behave like a free rotor. If, on the contrary, the rotor is supported in another way and the actuators are used to control its dynamic behavior, such matrices are not singular.

Equation (16.1) can be written with reference to the state space as

$$\dot{\mathbf{z}} = \mathbf{Az} + \mathbf{B}_c\mathbf{u}_c(t) + \mathbf{B}_e\mathbf{u}_e(t),$$ \hspace{1cm} (16.2)

where

- Vector $\mathbf{z}$ contains the $n$ complex state variables $\dot{\mathbf{x}}$ and $\mathbf{x}$.
- Vector $\mathbf{u}_c(t)$ contains the $r$ control input functions. Vector $\mathbf{u}_e$ contains all external inputs, related to nonrotating and rotating forces, which are usually (at least the rotating ones) functions of time.
- $\mathbf{A}$ is the dynamic matrix of the system,

$$\mathbf{A} = \begin{bmatrix} -\mathbf{M}^{-1}(\mathbf{C}_n + \mathbf{C}_r + \Omega\mathbf{G}) & -\mathbf{M}^{-1}(\mathbf{K} + \Omega\mathbf{C}_r) \\ \mathbf{I} & 0 \end{bmatrix}. \hspace{1cm} (16.3)$$

- Matrices $\mathbf{B}_c$ and $\mathbf{B}_e$ are the input gain matrices, respectively, for the control and external inputs.