The models seen in the previous two chapters allow us to understand the basic features of the dynamic behavior of rotating systems and the main differences between rotordynamics and standard structural dynamics. Although suited for qualitative studies, they are unable to give accurate quantitative predictions for the complicated rotors encountered in real-world machinery, owing to their complexity.

At the macroscopic level typical of machine design, flexible bodies can be modeled as continuous systems, and linear elastic rotors can be studied by writing the relevant differential equations describing the behavior of linear elastic continua. Such equations are partial derivative differential equations, containing the derivatives with respect to time, usually up to the second derivative, as well as derivatives with respect to space coordinates. The computational difficulties developing both from the differential equations and, even more, from the boundary conditions, can, however, be tackled only in a few simple cases (see Chapter 5). The solution of most problems encountered in engineering practice requires dealing with complex rotors, and the use of continuous models is, consequently, ruled out; the only feasible approach is the discretization of the continuum to obtain a discretized model written in terms of ordinary differential equations, containing only derivatives with respect to time.

The substitution of a continuous model, characterized by an infinite number of degrees of freedom, with a discrete one, sometimes with a very large but finite number of degrees of freedom, is usually referred to as discretization. This step is of primary importance in the solution of practical problems, because the accuracy of the results depends largely on the adequacy of
the discrete model to represent the actual system. The same discretization techniques widely used in structural dynamics can be applied in rotordynamics, provided that modifications aimed to introduce gyroscopic effects and other features typical of rotating systems are introduced.

After the discretization procedure has been applied, another problem, which is nowadays far less severe because of the growing power of computers, develops: The size of the discrete model, obtained through discretization, is usually very large, and in some cases, it may contain many hundreds, or even thousands, of degrees of freedom. Before the widespread use of computers, the solution of an eigenproblem containing matrices whose order was greater than a few units was very difficult and usually beyond actual possibilities. Many techniques were aimed at reducing to a minimum the size of the eigenproblem or transforming it into a form that could be solved using particular algorithms. The possibility of solving very large eigenproblems did actually change the basic approach to all problems in structural dynamics, and then also rotordynamics, making many popular methods obsolete.

The most common methods used in rotordynamics can be subdivided into two wide classes, the so-called lumped-parameters methods and the finite element method (FEM).

In the first case, the mass of the system (rotor and stator, if the latter is included in the system) is lumped into a certain number of rigid bodies (sometimes simply point masses) located at given stations in the deformable body. These lumped masses are then connected by massless fields that possess elastic and, sometimes, damping properties. Usually the properties of the fields are assumed to be uniform in space. Because the degrees of freedom of the lumped masses are used to describe the motion of the system, the model leads intuitively to a discrete system. Although the mass and gyroscopic matrices of such systems are easily obtained, it is often difficult to write the stiffness matrix, or, alternatively, the compliance matrix. To avoid such difficulty, together with that linked to the solution of large eigenproblems, an alternative approach can be followed. Instead of dealing with the system as a whole, the study can start at a certain station and proceed station by station using the so-called transfer matrices.

**Remark 4.1** Methods based on transfer matrices were very common in the past, because they could be worked out with tabular manual computations or implemented on very small computers. Their limitations are now making them yield to the finite element method.

A separate class can be assigned to the FEM. As the FEM is widely used in many fields of engineering analysis (structural, thermal, magnetic, etc.), only a short account will be given here. In the FEM, the body is subdivided into a number of regions, called finite elements, as opposed to the vanishingly small regions used in writing the differential equations.