Chapter 9

Curve Propagation, Level Set Methods and Grouping

N. Paragios

Abstract

Image segmentation and object extraction are among the most well-addressed topics in computational vision. In this chapter we present a comprehensive tutorial of level sets towards a flexible frame partition paradigm that could integrate edge-drive, regional-based and prior knowledge to object extraction. The central idea behind such an approach is to perform image partition through the propagation planar curves/surfaces. To this end, an objective function that aims to account for the expected visual properties of the object, impose certain smoothness constraints and encode prior knowledge on the geometric form of the object to be recovered is presented. Promising experimental results demonstrate the potential of such a method.

9.1 Introduction

Image segmentation has been a long term research initiative in computational vision. Extraction of prominent edges [381] and discontinuities between inhomogeneous image regions was the first attempt to address segmentation. Statistical methods that aim to separate regions according to their visual characteristics was an attempt to better address the problem [341], while the snake/active contour model [455] was a breakthrough in the the domain.

Objects are represented using parametric curves and segmentation is obtained through the deformation of such a curve towards the lowest potential of an objective function. Data-driven as well as internal smoothness terms were the components of such a function. Such a model refers to certain limitations like, the initial conditions, the parameterisation of the curve, the ability to cope with structures with multiple components, and the estimation of curve geometric properties.
Balloon models [204] where a first attempt to make the snake independent with respect to the initial conditions, while the use of regional terms forcing visual homogeneity [922] was a step further towards this direction. Prior knowledge was also introduced at some later point [756] through a learning stage of the snake coefficients. Geometric alternatives to snakes [152] like the geodesic active contour model [155] were an attempt to eliminate the parameterisation issue.

Curves are represented in an implicit manner through the level set method [618]. Such an approach can handle changes of topology and provide sufficient support to the estimation of the interface geometric properties. Furthermore, the use of such a space as an optimisation framework [917], and the integration of visual cues of different nature [622] made these approaches quite attractive to numerous domains [617]. One can also point recent successful attempts to introduce prior knowledge [513, 688] within the level set framework leading to efficient object extraction and tracking methods [689].

To conclude, curve propagation is an established technique to perform object extraction and image segmentation. Level set methods refer to a geometric alternative of curve propagation and have proven to be a quite efficient optimisation space to address numerous problems of computational vision. In this chapter, first we present the notion of curve optimisation in computer vision, then establishes a connection with the level set method and conclude with the introduction of ways to perform segmentation using edge-driven, statistical clustering and prior knowledge terms.

9.2 On the Propagation of Curves

Let us consider a planar curve $\Gamma : [0, 1] \rightarrow \mathbb{R} \times \mathbb{R}$ defined at a plane $\Omega$. The most general form of the snake model consists of:

$$E(\Gamma) = \int_0^1 (\alpha E_{\text{int}}(\Gamma(p)) + \beta E_{\text{img}}(\mathcal{I}(\Gamma(p))) + \gamma E_{\text{ext}}(\Gamma(p))) \, dp \quad (9.1)$$

where $\mathcal{I}$ is the input image, $E_{\text{int}} = [w_1|\Gamma'| + w_2|\Gamma''|]$ imposes smoothness constraints (smooth derivatives), $E_{\text{img}} = -|\nabla \mathcal{I}|$ makes the curve to be attracted from the image features (strong edges), $E_{\text{ext}}$ encodes either user interaction or prior knowledge and $\alpha, \beta, \gamma$ are coefficients that balance the importance of these terms.

The calculus of variations can be used to optimise such a cost function. To this end, a certain number of control points are selected along the curve, and their positions are updated according to the partial differential equation that is recovered through the derivation of $E(\Gamma)$ at a given control point of $\Gamma$. In the most general case a flow of the following nature is recovered:

$$\Gamma(p; \tau) = \left( \alpha F_{\text{gm}}(\Gamma) + \beta F_{\text{img}}(\mathcal{I}) + \gamma F_{\text{pr}}(\Gamma) \right) N \quad (9.2)$$