Homogenization of a Nonlinear Elliptic Boundary Value Problem Modelling Galvanic Interactions on a Heterogeneous Surface

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Summary. We study a nonlinear elliptic boundary value problem arising from electrochemistry. The boundary value problem occurs in the study of heterogeneous electrode surfaces. The boundary condition is of an exponential type and is normally associated with the names of Butler and Volmer and the notions of galvanic corrosion. We examine the questions of existence and uniqueness of solutions to this boundary value problem. We then treat the problem from the point of view of homogenization theory. The boundary condition has a periodic structure. We find a limiting or effective problem as the period approaches zero, along with a correction term and convergence estimates. We also do numerical experiments to investigate the behaviour of galvanic currents near the boundary as the period approaches zero.

Key words: galvanic corrosion, homogenization, nonlinear boundary condition

1 Introduction

Galvanic corrosion is a phenomenon caused by electrochemical interaction between different parts of the same surface. We study this phenomenon. A galvanic interaction occurs when galvanic current flows either between an electrode surface and a counterelectrode or between different parts of the same heterogeneous surface. In Figure 1(a) the silver strip is cathodic, and reduction takes place (Ag gains electrons.) Simultaneously oxidation takes place at the zinc strip, zinc loses electrons, and is said to be anodic. Zinc dissolves into the solution, the zinc electrode is being corroded and the electron flow is known as galvanic current.

In Figure 1(b), a similar oxidation-reduction reaction is taking place between different parts of the same surface. Here, in this paper, we consider a cylindrically shaped domain, and model the oxidation-reduction reaction occurring between different parts of our heterogeneous surface, i.e. the two dimensional
Fig. 1. (a) Zinc loses electrons to Silver, (b) A similar reaction occurs between different parts of the same surface base of our cylindrically shaped domain $\Omega$. The base, which we will refer to as $\Gamma$, contains a periodically regular arrangement of anodic islands in a cathodic plane. All the anodes are the same uniform material. The cathodic plane is also uniform in material (see Figure 2.)

![Diagram of an electrochemical reaction](image)

**Fig. 2.** The base of the cylinder is a heterogeneous surface.

The electrolytic voltage potential, $\phi$ satisfies the following nonlinear elliptic boundary value problem,

$$
\Delta \phi = 0 \text{ in } \Omega \\
-\frac{\partial \phi}{\partial n} = J_A[e^{\alpha_{aa}(\phi-V_A)} - e^{-\alpha_{ac}(\phi-V_A)}] \text{ on } \partial \Omega_A \\
-\frac{\partial \phi}{\partial n} = J_C[e^{\alpha_{ca}(\phi-V_C)} - e^{-\alpha_{cc}(\phi-V_C)}] \text{ on } \partial \Omega_C \\
-\frac{\partial \phi}{\partial n} = 0 \text{ on } \partial \Omega \setminus \{\partial \Omega_A \cup \partial \Omega_C\}
$$

The boundary condition is called the Butler-Volmer exponential boundary condition, where:

$$\alpha_{aa}, \alpha_{ac} = \text{anodic transfer coefficients, } \alpha_{aa} + \alpha_{ac} = 1$$