JÁNOS BOLYAI’S NEW FACE

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1. Introduction

János Bolyai’s (1802-1860) absolute and hyperbolic geometry made his name immortal in the history of mathematics. His life and scientific activity has been the topic of several studies. Bolyai’s impact on modern thinking has been presented by several scholarly works, which also show its various facets. As a mathematician, he is almost exclusively known for his achievements in geometry and the theory of complex numbers. The personal image of Bolyai, most people were familiar with earlier, had mostly been influenced by some literary works, dramatizing his irreconcilable relationship with his father and his disharmony with the people around him.

Bolyai’s many thousand-page legacy of manuscripts that has been brought to light by the research of recent years calls for a re-evaluation of his achievements as a scientist and his personality.

The Appendix [2] is not the only work János Bolyai left to posterity. Even after the completion of his exceptional work, he continued his research and put down his ideas. This resulted in his vast legacy of manuscripts, fourteen thousand pages of which are in the Teleki-Bolyai Library of Marosvásárhely. The notes contain those mathematical theorems, which prove not only that Bolyai brought a significant contribution to geometry, but also that he reached valuable conclusions in other mathematical fields.

First of all, we can assert that in the past our knowledge of his results in mathematics was rather superficial. Bolyai excels not only with his achievements in geometry, but also with his research in several other mathematical fields. His accomplishments in number theory and algebra are most astounding.
2. János Bolyai’s Research on Number Theory

All Bolyai-monographs unanimously assert: although János Bolyai tried his hand at a few problems in number theory, his investigations were not particularly successful. However, his manuscripts attest the opposite of all this. Bolyai had a keen interest in questions of number theory and he had several original ideas, with which he preceded many other mathematicians of later ages.

Among the very first theorems found in Bolyai’s legacy is the following: If \( p \) and \( q \) are prime numbers, and \( a \) is an integer divisible neither by \( p \) nor by \( q \), and if \( a^{p-1} \equiv 1 \) (mod \( q \)) and \( a^{q-1} \equiv 1 \) (mod \( p \)), then

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a^{pq-1} \equiv 1 \pmod{pq}.
\]

We can readily observe that it is the same theorem which James Hopwood Jeans (1877-1946) published decades later in 1898. Since we can definitely affirm that this relation was first recognized and demonstrated