INTRODUCTION

Quantum theory is connected especially with the names Planck, Bohr, Heisenberg, Pauli, and Dirac. The quantum revolution describes our deepest insight, so far, into the physical structure of nature. It is comparable only with the Copernican revolution, switching from a finally oriented anthropocentric description of physical phenomena to one using general laws with initial or boundary conditions, connected with the names Kepler, Galileo, and Newton, or with the change from tangible mass points as basic structures to Faraday's and Maxwell's field concepts and, shortly before quantum theory, with the relativization of space and time by the lonely genius Einstein.

In retrospect, the label "quantum" or, as adjective, "quantal," is too weak to characterize the extent of the revolution involved in abandoning the classical theory as a basic epistemological framework for physics. The word "quantal" – in contrast to the assumed classical "continuous" ("natura non facit saltus") – was motivated by the finite jumps and the discreteness as seen, for example, in the photoelectric effect or in the spectral lines for atoms or in the discrete split of atomic rays in Stern-Gerlach experiments.

One has to distinguish in quantum theory between two kinds of "jumps": First, the quantum structure relies on the noncommutativity of operations, e.g., of the not commuting position-momentum operator pair $[i\mathbf{p}, \mathbf{x}] = \hbar$, with a nontrivial quantum $\hbar$ (Planck's constant) or of the not anticommuting conjugate operator pair of an electron-positron field $\{\overline{\mathbf{\Psi}}(\vec{y}), \mathbf{\Psi}(\vec{x})\} = \hbar\gamma^0\delta(\vec{x} - \vec{y})$. Second, there are the jumps, characterized by integers. These jumps, as seen in the atomic spectral lines, were the starting point of quantum theory. However, after the dust has settled, they cannot be addressed as the revolutionary characteristics of quantum theory: Integers characterize compact operation groups. Take a circle, say a closed rubber string, cut it, wind it around your wrist, and glue both ends together again; the number of possible windings is always an integer. Does rubber band winding characterize quantum theory? The rubber band stands for the circle, parametrizing the compact Lie group $\mathbf{U}(1) = \exp i\mathbb{R}$ or the isomorphic group $\mathbf{SO}(2)$ with the rotations around one space axis. The irreducible representations of the circle (1-dimensional torus), as realized by the different rubber band windings and thus of all compact Lie groups involving higher-dimensional tori, come with integer winding numbers, "quantum numbers" in the narrow sense. Since bound waves in quantum mechanics are related to compact representations of the noncompact time translation group $\mathbb{R}$, they give rise to integer-related discrete (rational)

quantum jumps. The same situation occurs for spin, which is related to the 3-dimensional position rotations, parametrizable by the compact volume of a sphere. However, in addition to these discrete jumps (integer winding numbers $z \in \mathbb{Z}$) continuous quantum numbers can also occur, e.g., real energies $E \in \mathbb{R}$ or momenta $\vec{q} \in \mathbb{R}^3$, or, apparently, the particle masses $m^2 \in \mathbb{R}_+$ from a continuous spectrum as eigenvalues or invariants for representations of time and space translations. Continuous numbers require operations with noncompact action groups, whereas compact groups come with rational ("quantum") numbers.

At the core of quantum theory is the relativization of the ontic structures in contrast to the absolute ontology in classical theories, e.g., of the position of mass points or of the spin direction of particles. The appropriate characterization "quantum relativity" alludes to the relativity of time and space. A quantum description starts from practic stuctures, e.g., from translations or rotations. Quantum theory describes operations with the dynamics itself an operation. Quantum theory is operation theory. A classical ontology requires a projection of the nonabelian operational framework to an abelian substructure. In a classical description, objects are primary with interactions between them as a secondary structure. In a quantum description the hierarchy is reversed: objects arise as eigenvectors of operations.

Appropriate questions in quantum theory ask for operations: What is the operational meaning of spin and mass of a particle? Invariants for rotations and spacetime translations. What is the operational meaning of a Coulomb and Yukawa potential? Representation distributions, 2-sphere spreads of position translations. What is the operational meaning of a gauge coupling constant? The relative normalization of the gauge–transformation–inducing operational Lie algebra in the Lorentz Lie algebra. What is the operational meaning of a Feynman propagator? Matrix elements of spacetime translation representations, unitary for on-shell contributions.

And one may ask even about quite specific structures: What is the operational meaning of cosines and exponentials, of Bessel and Macdonald functions, or of Laguerre polynomials, etc.? Representation coefficients of specific operations. With respect to a formulation of physics by special functions arising as solutions of "special differential equations," e.g., equations of motion in time and space, there is a unified view, initiated by Wigner and elaborated in exhaustive encyclopedic detail by Vilenkin, who writes in the introduction of his subject–related book, "a really unified view on the theory of the basic classes of special functions ... was established by employing the considerations that belong to a field of mathematics seemingly quite far from the subject under consideration, the theory of representations of Lie groups." Essentially all physically relevant special functions arise as coefficients of Lie group representations. Therefore in the following, Lie operations are of paramount importance.

Weyl was the first to connect with each other, basically and in a systematic form, "The theory of groups and quantum mechanics" in his like-named book. Wigner especially proceeded to extend the group–theoretic method in mathematical detail to relativistic quantum theory.