4

MASSIVE PARTICLE QUANTUM FIELDS

With the experience of quantum mechanics on the one hand, where time comes as a real parameter and position as an operator in a Bose quantum algebra $\mathbf{Q}_-(\mathbb{C}^2)$, and special relativity on the other hand, where time and position constitute Minkowski spacetime, one may expect a relativistic quantum structure with both time and position as operators. However both time and position $x = (t, \vec{x})$ are used not as operators, but "only" as real parameters for the translation behavior of relativistic fields. The value space of the fields carries the quantum degrees of freedom, with the example of a free scalar quantum field $\mathbf{\Phi}$ for massive neutral particles:

$$\begin{array}{c} \boxed{\begin{array}{c}\text{time}\\ \text{translations}\\ t \in \mathbb{R}\end{array}} \\ \downarrow \\ \boxed{\mathbf{x}(t)} \end{array} \qquad\qquad \begin{array}{c} \boxed{\begin{array}{c}\text{spacetime}\\ \text{translations}\\ x \in \mathbb{R}^4\end{array}} \\ \downarrow \\ \boxed{\mathbf{\Phi}(x)} \end{array}$$

$$\text{quantum mechanics} \quad \begin{array}{rcl} \mathbb{R} & \hookrightarrow & \mathbb{R}^4 \\ \mathbf{Q}_-(\mathbf{W}) & \hookrightarrow & \mathbf{Q}_-(\mathbf{W}^{\mathcal{Y}^3}) \end{array} \quad \text{quantum field theory}$$

The quantum algebras come as value spaces for mappings $\{\mathcal{Y}^3 \longrightarrow W\}$ of the energy-momentum hyperboloids $q^2 = m^2 > 0$ for free particles. For each momentum $\vec{q}$ and spin J, there is a quantum algebra over a representation space $W \supseteq \mathbb{C}^{1+2J}$ for spin and chargelike operations. In retrospect, quantum mechanics is characterizable by quantum orbits of time with the quantum structure implemented by position. The quantum structure of the orbits of relativistic spacetime is implemented by field degrees of freedom, e.g., by spin, electromagnetic charge, isospin, etc.

Free particles are characterized by irreducible Hilbert representations of the Poincaré group $\mathbf{SL}(\mathbb{C}^2) \,\vec{\times}\, \mathbb{R}^4$. The infinite-dimensional representations for particles are induced by and embed Hilbert representations of spacetime translations $\mathbb{R}^4$ and of position rotations, i.e., of spin $\mathbf{SU}(2)$ for massive particles $m^2 > 0$ (for convenience $m = |m|$ throughout this chapter) and of circularity (helicity, polarization) $\mathbf{SO}(2)$ for massless particles (chapter "Massless

Quantum Fields"). More mathematical details are discussed in the chapter "Harmonic Analysis."

Particles are embedded into quantum fields. The Feynman propagator for a particle field describes its spacetime behavior. It has two parts which are analogous to the two wave function types in quantum mechanics (chapter "The Kepler Factor"): For kinetic energy $E - V_0 = \frac{\vec{q}^2}{2} > 0$, there arise scattering waves $\frac{\sin|\vec{q}|r}{r}$, whereas bound waves $e^{-|Q|r}$ come with binding energy $E - V_0 = -\frac{Q^2}{2} < 0$. In a Feynman propagator, the scattering part is embedded into the Fock form function of the quantization opposite commutators. It involves matrix elements of Hilbert representations of the translations $\mathbb{R}^4$. This part describes free particles: on-shell with kinetic energy $q_0^2 - m^2 = \vec{q}^2 > 0$. The relativistic correspondence to the nonrelativistic bound waves is the $\epsilon(x_0)$-multiplied quantization (anti-) commutator distribution, which contains off-shell contributions ("virtual particles"), $q^2 \neq m^2$. The embedded Yukawa interactions $\frac{e^{-|Q|r}}{r}$ and forces are distributions (2-sphere spreads) of representation coefficients of position with imaginary "momentum" $q_0^2 - m^2 = -Q^2 < 0$ as eigenvalues, the analogue to the nonrelativistic binding energy. These "virtual particle" contributions have small-distance $r = 0$ singularities; they are not representation coefficients of the spacetime translations.

In addition to the translation properties, i.e., the invariant mass, and the energy-momenta eigenvalues, particle fields have homogeneous Lorentz transformation properties. Massive particle fields come with decompositions of Minkowski translations into time and position translations, induced by a rest system of the field embedded particle and determined up to rotations $\mathbf{SO}(3) \cong \mathbf{SU}(2)/\mathbb{I}(2)$. The $\mathbf{SU}(2)$-representations determine the spin of the particle.

Relativistic quantum fields for massive particles have particle degrees of freedom only. Their Hilbert representations allow a complete probability interpretation. This is in contrast to massless fields (chapter "Massless Quantum Fields").

The complex representations of the real-spacetime-related groups are in unitary groups that contain $\mathbf{U}(1)$-phase groups. An "internal" $\mathbf{U}(1)$-group in addition to the translations representing $\mathbf{U}(1)$ describes particles and antiparticles. For example, the Dirac representation of the Lorentz group in the anticonjugation group $\mathbf{SL}(\mathbb{C}^2) \longrightarrow \mathbf{U}(2,2)$ uses the probability-inducing $\mathbf{U}(\mathbf{1}_4)$ for the translations and the additional relative phase in $\mathbf{SU}(2,2) \supset \mathbf{U}(\mathbf{1}_2)_3 \times \mathbf{SL}(\mathbb{C}^2)$ for an internal "chargelike" $\mathbf{U}(1)$. With the exception of Majorana structures, all half-integer spin particles have an additional internal $\mathbf{U}(1)$-charge, arising, e.g., for neutrino-antineutrino as fermion number and for electron-positron as electromagnetic charge.

For relativistic quantum fields, the Lie algebras of the external Poincaré group and of the internal operations are implemented by position integrals of generator distributions, their currents, which are written with the quantization opposite commutators.

After a review of the $\mathbf{U}(1)$-representations for the time group $\mathbf{D}(1) \cong \mathbb{R}$ in quantum algebras, i.e., of the harmonic Fermi and Bose quantum oscilla-