5

MASSLESS QUANTUM FIELDS

Massless particle use, for their irreducible Hilbert representation of the Poincaré group, spacetime decompositions into time and position translations with one distinguished rotational axis. The position axis is fixed by the momentum direction of the never-resting particle and determined up to axial rotations $\mathbf{SO}(2)$ whose representations determine the circularity (helicity, polarization) of the particle. Axial rotations act on particle pairs with opposite circularity $\pm 2J \in \mathbb{Z}$. Strictly speaking, massless particles have no $\mathbf{SU}(2)$-spin; they have $\mathbf{SO}(2)$-polarization or helicity.

An axial rotation fixgroup $\mathbf{SO}(2)$ in a noncompact Euclidean fixgroup with two boosts $\mathbf{SO}(2) \vec{\times} \mathbb{R}^2 \subset \mathbf{SO}_0(1,3)$ gives additional structures compared with the massive case and a rotation fixgroup in the Lorentz group $\mathbf{SO}(3) \subset \mathbf{SO}_0(1,3)$: With the embedding of particles with axial rotation $\mathbf{SO}(2)$ properties into quantum fields with finite-dimensional Lorentz group representations, there can arise translation representations not only in the probability group $\mathbf{U}(1)$, but also in the noncompact group $\mathbf{U}(1,1)$ (indefinite metric). Massless quantum particle fields can have degrees of freedom without probabilistic particle interpretation, i.e., without state vectors in a Hilbert space. Nonparticle degrees of freedom in relativistic fields describe genuine interactions, e.g., the Coulomb interaction, which comes in addition to the two photons in the four components of an electromagnetic vector field.

The spacetime translation development of a mass-zero vector field involves eigenvectors (particles) and lightcone-related nilvectors. The eigenvector property is expressible by a trivial action of the nil-Hamiltonian, which in a quantum theory is equivalent to a trivial action of the nilquadratic Becchi-Rouet-Stora charge, constructed with the probability interpretation securing Fermi Fadeev-Popov scalar fields. The classical limit of the BRS-transformation gives Lie algebra transformations with spacetime-dependent parameters, which replace the Fadeev-Popov fields and are familiar as "gauge transformations." The translation eigenvectors with trivial BRS-charge are "gauge invariant."

After a review of indefinite unitary time translations as implemented in quantum algebras and the definition of a Hilbert space for translation

eigenvectors, the relativistic embedding of massless particles is given with definite and indefinite metric degrees of freedom in their quantum fields.

5.1 Noncompact Time Representations in Quantum Algebras

The nondecomposable complex 2-dimensional time representations with invariant energy (frequency) m and basis-dependent nilconstant ν on a complex 2-dimensional vector space are in the noncompact group $\mathbf{U}(1,1)$ (chapter "Time Representations"). They are faithful:

$$\begin{aligned} \mathbb{R} \ni t \longmapsto\ & e^{imt}\begin{pmatrix} 1 & i\nu t \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & \nu\frac{d}{dm} \\ 0 & 1 \end{pmatrix} e^{imt} \in \mathbf{U}(1,1) \subset \mathbf{GL}(\mathbb{C}^2), \\ & m, \nu \in \mathbb{R}, \quad \nu \neq 0. \end{aligned}$$

Dual bases of the representation spaces

$$\mathrm{b}, \mathrm{g} \in V \cong \mathbb{C}^2 \cong V^T \ni \mathrm{g}^\times, \mathrm{b}^\times$$

have the equations of motion, with $d_t = \frac{d}{dt}$, and time orbits

$$d_t\begin{pmatrix} \mathrm{b} \\ \mathrm{g} \end{pmatrix} = i\begin{pmatrix} m & \nu \\ 0 & m \end{pmatrix}\begin{pmatrix} \mathrm{b} \\ \mathrm{g} \end{pmatrix}, \qquad d_t(\mathrm{g}^\times, \mathrm{b}^\times) = -i(\mathrm{g}^\times, \mathrm{b}^\times)\begin{pmatrix} m & \nu \\ 0 & m \end{pmatrix},$$

$$\begin{pmatrix} \mathrm{b} \\ \mathrm{g} \end{pmatrix}(t) = e^{imt}\begin{pmatrix} 1 & i\nu t \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \mathrm{b} \\ \mathrm{g} \end{pmatrix}, \qquad (\mathrm{g}^\times, \mathrm{b}^\times)(t) = (\mathrm{g}^\times, \mathrm{b}^\times)e^{-imt}\begin{pmatrix} 1 & -i\nu t \\ 0 & 1 \end{pmatrix}.$$

Only g and $\mathrm{g}^\times$ are time development eigenvectors, the letters $\{\mathrm{g}, \mathbf{g}, \mathbf{G}, \gamma\}$ stand for "good" (eigenvectors) and $\{\mathrm{b}, \mathbf{b}, \mathbf{B}, \beta\}$ for "bad" (nilvectors).

For trivial nilconstant $\nu = 0$ there remain two $\mathbf{U}(1)$-representations, which are compatible with $\mathbf{U}(1,1)$-conjugation $\times$ and $\mathbf{U}(1)$-conjugation $\star$:

$$\nu = 0 : \left\{ \begin{array}{cl} (\mathrm{b}, \mathrm{g}) & = (\mathrm{a}, \mathrm{u}), \\ (\mathrm{b}^\times, \mathrm{g}^\times) & = (\mathrm{a}^\times, \mathrm{u}^\times) = (\mathrm{u}^\star, \mathrm{a}^\star). \end{array} \right.$$

The notation $\{\mathrm{b}, \mathrm{g}\}$ will also be used for $\nu = 0$ with $\mathbf{U}(1,1)$-conjugation.

A $\mathbf{U}(1,1)$-symmetric basis of $\mathbf{V} = V \oplus V^T \cong \mathbb{C}^4$

$$\mathbf{b}_+ = \tfrac{\mathrm{b}+\mathrm{b}^\times}{\sqrt{2}}, \quad \mathbf{b}_- = \tfrac{\mathrm{b}-\mathrm{b}^\times}{i\sqrt{2}}, \quad \mathbf{g}_+ = \tfrac{\mathrm{g}+\mathrm{g}^\times}{\sqrt{2}}, \quad \mathbf{g}_- = \tfrac{\mathrm{g}-\mathrm{g}^\times}{i\sqrt{2}},$$

is acted on by a real time representation

$$\begin{aligned} d_t(\mathbf{g}_+, \mathbf{g}_-, \mathbf{b}_+, \mathbf{b}_-) &= (\mathbf{g}_+, \mathbf{g}_-, \mathbf{b}_+, \mathbf{b}_-)h_1(m, \nu), \\ h_1(m, \nu) &= \left(\begin{array}{cc|cc} 0 & m & 0 & \nu \\ -m & 0 & -\nu & 0 \\ \hline 0 & 0 & 0 & m \\ 0 & 0 & -m & 0 \end{array}\right) = \begin{pmatrix} m & \nu \\ 0 & m \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 1 & \nu\frac{d}{dm} \\ 0 & 1 \end{pmatrix} \otimes h_0(m), \quad h_0(m) = m\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \end{aligned}$$