6

GAUGE INTERACTIONS

With the work of Weyl and London on gauge theories, Maxwell's equations for Faraday's electromagnetic field concepts proved to be a theory of phase $\mathbf{U}(1)$-operations that act, compatibly with spacetime translations, on complex representation spaces. In quantum electrodynamics the electromagnetic $\mathbf{U}(1)$ is implemented by the electromagnetic potential (field) interacting with Dirac fields for electrons and positrons. The standard model of the electroweak and strong interactions for lepton and quark quantum fields embeds quantum electrodynamics into a representation theory for the compact internal action groups $\mathbf{U}(1)$ (hypercharge), $\mathbf{SU}(2)$ (isospin) and $\mathbf{SU}(3)$ (color), implemented by twelve gauge fields acting on left- and right-handed Weyl fields and, for nonabelian groups, on themselves:

$$\boxed{\begin{array}{c}\text{electrostatics}\\ \mathbf{SO}(3)\ \vec{\times}\,\mathbb{R}^3\end{array}} \hookrightarrow \boxed{\begin{array}{c}\text{electrodynamics}\\ \mathbf{SO}_0(1,3)\ \vec{\times}\,\mathbb{R}^4\end{array}}$$

$$\hookrightarrow \boxed{\begin{array}{c}\text{quantum electrodynamics}\\ \mathbf{U}(1)\times[\mathbf{SO}_0(1,3)\ \vec{\times}\,\mathbb{R}^4]\end{array}} \hookrightarrow \boxed{\begin{array}{c}\text{standard gauge interactions}\\ \mathbf{U}(2\times 3)\times[\mathbf{SO}_0(1,3)\ \vec{\times}\,\mathbb{R}^4]\end{array}}$$

It is remarkable that each of the incomplete theories shows its own esthetics and beauty.

All spacetime translations have to take into account the orientation of the internally acting group, there is no spacetime translation without internal group action. In quantum field theory, Lorentz compatible distributions of Lie algebra representations define currents (chapter "Massive Particle Quantum Fields'). The representation of a Lie algebra on a vector space is a power-three tensor whose spacetime distribution comes as a product of the current with the gauge field. Such a power three tensor constitutes a gauge interaction vertex for a field theory. This is used for the real 12-parametric standard model Lie symmetry with its $(1+3+8)$ gauge fields.

The internal action groups for hypercharge, isospin, and color come in centrally correlated representations, the eigenvalue of the abelian hypercharge $\mathbf{U}(1)$-action is related to the center representation of the nonabelian isospin-color group $\mathbf{SU}(2)\times\mathbf{SU}(3)$. For example, isospin doublets and color triplets

come with hypercharge factors $\frac{1}{2}$ and $\frac{1}{3}$, a doublet-triplet quark with hypercharge $\frac{1}{6}$.

From the 12-parametric internal symmetry operations for interactions there remains only an electromagnetic $\mathbf{U}(1)$-symmetry for particles. The isospin $\mathbf{SU}(2)$-symmetry is broken ("bleached"), it leaves its trace in particle multiplicities. This is in contrast to color $\mathbf{SU}(3)$, where experiments show only trivial color representations for particles which is interpreted as color confinement. A simultaneous diagonalization of the rank $1+1+2=4$ centrally correlated symmetry structure of the interaction is possible for a maximal abelian subgroup that is trivial either for isospin $\mathbf{SU}(2)$ or for color $\mathbf{SU}(3)$. Taking a color-trivial maximal diagonalization, the electroweak $\mathbf{U}(2)$-operations require a projection to an electromagnetic $\mathbf{U}(1)$ Cartan subgroup, correlating hypercharge and isospin, as remaining internal particle symmetry group. In the standard model, this projection (electroweak symmetry breakdown) is effected by a ground state, degenerate with the Goldstone manifold $\mathbf{U}(2)/\mathbf{U}(1)$ and implemented by a scalar field (Higgs field).

After a short review of classical and quantum electrodynamics, its embedding into the standard model of electroweak and strong interactions is discussed together with the ground state induced rearrangement of the interactions to the particle language.

6.1 Classical Maxwell Equations

Experiences with amber (electron) and stones from Magnesia (Greek town in Asia Minor) and experiments have shown the existence of "nonmechanical" interactions, especially nongravitational ones, which, in today's language, cannot be related to Poincaré group, i.e., external, operations. On the "spacetime screen," i.e., with each spacetime translation, there also act "internal" operations. The electric and magnetic interactions were taken as a first hint for "charge" related operation groups.

In the beginning, it was enough to characterize these properties (eigenvalues) by an *electric charge* Q, first measured[1] in an ad hoc unit, e.g., $[Q] = \mathrm{C}$ (coulomb), introduced for a dimensional grading in addition to units for length, time and mass, which can be measured with, e.g., the ad hoc human order of magnitude units $[L] = \mathrm{m}$ (meter), $[T] = \mathrm{s}$ (second), and $[M] = \mathrm{kg}$ (kilogram). In the course of this chapter an independent charge unit will be replaced by an $[L], [T], [M]$-derived unit and two of the remaining "human" units will be replaced by natural or structurally intrinsic units.

Charges change in time $t \longmapsto Q(t)$ (time orbits), which leads to the definition of an *electric current* $I(t)$:

$$I = -d_t Q \text{ with } [I] = \tfrac{[Q]}{\mathrm{s}}.$$

Now the historical transition to a framework with position-dependent fields: It appeared possible to distribute charge in position with an $\mathbf{SO}(3)$-scalar

[1] For $[a]$ read "unit of a,"