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Mixing, Nonparametric and Functional Statistics

Modelling dependence is of great interest in statistics. This comes mainly from the fact that it opens the door for application involving time series. Naturally, this question should be attacked in the nonparametric functional data context of this book. These statistical motivations will not be discussed in this chapter but later on in Chapter 12. The aim of this chapter is to recall some basic definitions, to discuss briefly the existing literature in finite dimensional setting and to introduce notations and general advances in the functional setting. This will prepare the way for the theoretical advances in nonparametric functional statistics for dependent samples that will be presented in Chapter 11.

10.1 Mixing: a Short Introduction

Mixing conditions are usual structures for modelling dependence for a sequence of random variables. It is out of our scope to present an exhaustive discussion on this point. A good overview of probabilistic knowledges on this notion can be found for instance in [Y92] (other references could be [B86] or [D95]). The reader may be interested also in the monographs by [GHSV89], [Y94], or [B98] which are centered both on the mixing structures themselves and on their interest for nonparametric statistics. The monographs [Y93a] and [Y93b] discuss the interest of mixing for statistical settings other than nonparametric estimation. Before going into the use of mixing notions in nonparametric statistics let us first recall some definitions and fix some notations. In our book we focus on the α-mixing (or strong mixing) notion, which is one of the most general among the different mixing structures introduced in the literature (see for instance [RI87] or Chapter 1 in [Y94] for definitions of various other mixing structures and links between them). This strong mixing notion is defined in the following way.

Let $(\xi_n)_{n\in\mathbb{Z}}$ be a sequence of random variables defined on some probabilistic space $(\Omega, \mathcal{A}, P)$ and taking values in some space $(\Omega', \mathcal{A}')$. Let us denote, for

$-\infty \leq j \leq k \leq +\infty$, by $\mathcal{A}_j^k$ the σ-algebra generated by the random variables $(\xi_s, j \leq s \leq k)$. The strong mixing coefficients are defined to be the following quantities:

$$\alpha(n) \; = \; \sup_k \; \sup_{A \in \mathcal{A}_{-\infty}^k} \; \sup_{B \in \mathcal{A}_{n+k}^{+\infty}} \left| P(A \cap B) \; - \; P(A)P(B) \right| .$$

The following definition of mixing processes was originally introduced by [R56].

Definition 10.1. *The sequence* $(\xi_n)_{n \in \mathbb{Z}}$ *is said to be* α*-mixing (or strongly mixing), if*

$$\lim_{n \to \infty} \alpha(n) = 0.$$

In the remainder of this book, in order to simplify the presentation of the results and not to mask our main purpose, we will mainly consider both of the following subclasses of mixing sequences:

Definition 10.2. *The sequence* $(\xi_n)_{n \in \mathbb{Z}}$ *is said to be arithmetically (or equivalently algebraically)* α*-mixing with rate* $a > 0$ *if*

$$\exists C > 0, \; \alpha(n) \; \leq \; C \, n^{-a}.$$

It is called geometrically α*-mixing if*

$$\exists C > 0, \; \exists t \in (0, 1), \; \alpha(n) \; \leq \; C \, t^n.$$

10.2 The Finite-Dimensional Setting: a Short Overview

Since a long time ago, and starting with earlier advances provided for instance by [R69] or [Ro69], mixing assumptions have been widely used in nonparametric statistics involving finite dimensional random variables. It is really out of purpose to make here a presentation of the very plentiful literature existing in this field, but one could reasonably say that almost all the results stated in finite dimensional nonparametric statistics for i.i.d. variables have been extended to mixing data. Key previous papers in this direction were those concerning regression estimation from mixing samples (see for instance [R83] and [C84]), but now extensions of nonparametric methods for mixing variables are available in most problems involving density, hazard, conditional density, conditional c.d.f., spectral density, etc. The reader who would be interested in having a good overview of all the literature should look at some among