11

Some Selected Asymptotics

The aim of this chapter is to give extensions to dependent data of several asymptotic results presented in Parts II and III of this book. This chapter is exclusively theoretical, while statistical applications in time series analysis and computational issues are reported in Chapter 12. Sections 11.2, 11.3 and 11.4 will concern the question of predicting some real-valued random response given a functional explanatory variable. Along these previous sections, some auxiliary results about conditional distribution estimation are stated. Section 11.5 is specially devoted to the presentation of these results. Then, Section 11.6 will be concerned with the discrimination problem. Our main goal in this chapter is to show how the dependence is acting on the asymptotic behaviour of the nonparametric functional methods. So, we have decided to present the results by emphasizing (on the hypothesis, as well as on the statement or on the proofs of the results) what is new with α-mixing variables compared with the standard i.i.d. case.

11.1 Introduction

There are always great motivations for studying the behaviour of any statistical method when the usual independence condition on the statistical sample is relaxed. The main reason for this comes from the wish to consider statistical problems involving time series. Of course, this question also occurs with nonparametric functional methods. The aim of this chapter is to provide some theoretical supports about the behaviour on dependent samples of the methods proposed in previous parts of this book. This will be done by means of some almost complete convergence results under mixing dependence modelling that will show the good theoretical behavior of the kernel methods for functional dependent statistical samples. A similar idea, but from a practical point of view, will be supported in Chapter 12.

The chapter is organized as follows: Sections 11.2, 11.3 and 11.4 will cover the question of predicting some real valued random response given a functional

explanatory variable. Each of these three sections will attack this problem by means respectively of regression estimation, of functional conditional mode estimation and of functional conditional quantile estimation. Nonparametric estimation is carried out by means of kernel methods, and complete convergence type results will be stated under some strong mixing assumption on the statistical sample. Section 11.5 will present asymptotic results for kernel estimation of conditional density and c.d.f. under mixing assumption. In other words, these four sections will show how the results stated in Chapter 6 for i.i.d. variables remain true in dependent situations. In the same spirit, Section 11.6 is concerned with the discrimination problem. By means of almost complete type results, it will be shown how the kernel supervised classification method behaves asymptotically for discriminating a sample of mixing curves. In other words, Section 11.6 will extend to mixing samples the results stated in Chapter 8 for i.i.d. variables. As for the independent case, the convergence properties are obtained under continuity-type models whereas rates of convergence need Lipschitz-type models.

Our main goal in this chapter is to show how the dependence is acting on the asymptotic behaviour of the nonparametric functional methods. So, we have decided to present our results by emphasizing what is new with mixing variables compared with the standard i.i.d. case. This concerns the presentation of the hypothesis and the statements of the main results. We will highlight how the mixing coefficients are changing (or not) the rates of convergence of the estimates. In the same spirit, our proofs will be rather short (but complete), since we only have to pay attention to the parts for which the dependence structure has some effect (basically the covariance terms in our asymptotic expansions). The other parts (basically bias and variance terms) are behaving as for i.i.d. variables and they will be quickly treated just by refering to previous parts of this book. Of course, such a synthetic presentation has the drawback of obliging the reader to keep in mind previous chapters. However, it has the great advantage of highlighting the influence of the dependence structure on the rates of convergence. It also helps avoid useless repetitions of tedious calculous and notations.

11.2 Prediction with Kernel Regression Estimator

11.2.1 Introduction and Notation

We wish to attack the same problem as described in Chapter 5 but under some dependence assumption on the statistical variables. Precisely, we have to predict a scalar response Y from a functional predictor $\boldsymbol{\mathcal{X}}$ and we will use the nonlinear regression operator r defined by:

$$r(\chi) = \mathbb{E}(Y|\boldsymbol{\mathcal{X}} = \chi). \tag{11.1}$$

Recall that $\boldsymbol{\mathcal{X}}$ is a functional random variable valued in some semi-metric space (E, d), Y is a real random variable, and χ is a fixed element of E.