TWO-COMPONENT BOSE-EINSTEIN CONDENSATES IN OPTICAL LATTICES
Modulational Instability and Soliton Generation

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Abstract Coupled nonlinear Schrödinger equations (CNLS) with an external elliptic function potential model a quasi one-dimensional interacting two-component Bose-Einstein condensate trapped in a standing light wave. New stationary solutions of the CNLS with a periodic potential are derived and interpreted as exact Bloch states at the edge of the Brillouin zone. The modulationally unstable solutions lead to formation of localized ground states of the coupled BEC system.

Keywords: Bose-Einstein condensate, optical lattices, modulational instability, soliton generation.

1. Introduction

Recent experiments on dilute-gas Bose-Einstein condensates (BEC’s) have generated great interest both from theoretical and experimental...
points of view [1]. At ultra-low temperatures the mean-field description for the macroscopic BEC wave-function is constructed using Hartree-Fock approximation and results in the Gross-Pitaevskii (GP) equation [1]. The latter one reduces to the one-dimensional (1D) nonlinear Schrödinger (NLS) equation with an external potential, in particular, when the transverse dimensions of the condensate are much less than its healing length and its longitudinal dimension is of order or much longer than the healing length (see e.g. [2, 3]). This is termed the quasi-one dimensional (quasi-1D) regime of the GP equation. In this regime BECs remain phase–coherent, and the governing equations are one-dimensional. Several families of stationary solutions for the cubic NLS with an elliptic function potential were presented in Refs [4, 6]; their stability was examined using analytic and numerical methods [6, 7, 8, 9, 10].

The two-component BEC’s is described by GP equations, which in the quasi-1D regime, reduce to coupled nonlinear Schrödinger (CNLS) equations with an external potential [11, 12].

Below we study the stationary solutions of the CNLS with an external potential. Several cases of explicit solutions in terms of elliptic functions are analyzed and their stability properties are studied numerically. We derive a set of stationary solutions with trivial and non trivial phases; some of them were also analyzed independently in Ref. [13]. We extend their results by investigating in more details the solutions of CNLS whose components are expressed through different elliptic functions, see also Section 6. We investigate the possibility that these solutions taken as initial states, may generate localized matter waves (solitons).

2. Basic equations

At very low temperatures, when the mean field approximation is applicable, the evolution of two interacting BEC’s is described by two coupled GP equations:

$$i\hbar \frac{\partial \Psi_j}{\partial t} = \left[ -\frac{\hbar^2}{2m} \nabla^2 + V_j(r) + \frac{4\pi \hbar^2}{m} \sum_{l=1,2} a_{jl} |\Psi_l|^2 \right] \Psi_j \quad (1)$$

($j = 1, 2$), see [11, 12]. Here atomic masses of both components are assumed to be equal, $V_j(r)$’s are external trap potentials, and $a_{ij}$ are the scattering lengths of the respective atomic interactions (other notations are standard). If $V_j$ consist of superposition of a magnetic trap providing cigar shape of the condensate (elongated, along the $x$-axis) and an optical trap inducing a periodic lattice potential along the $x$-axes we have:

$$V_j(r) = \frac{m}{2} \omega_j^2 [x^2 + y^2 + z^2] + U(\kappa x), \quad U(\kappa x) = U(\kappa (x + L)). \quad (2)$$