Chapter 2

BACKGROUND MATERIAL

The beginning of a framework: mastering the language.
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It is assumed that the reader has an elementary knowledge of vector analysis, differential equations and (geophysical) fluid dynamics. To make reading through the chapters 5-7 more easy, some background material is included in this chapter. The general equations of motion are presented in section 2.1; this serves also to introduce the notation used in the book. There are many textbooks available where these equations are derived and discussed (Batchelor, 1974; Pedlosky, 1987; Cushman-Roisin, 1994). In geophysical fluid dynamics and dynamical oceanography, many results are interpreted in terms of vorticity transport within the flow. In the sections 2.2 and 2.3, the mechanisms of vorticity transport and the concept of potential vorticity are illustrated by using simple examples. These examples serve as a reference for the terminology used in later chapters. The last piece of background material is elementary hydrodynamic stability theory. In section 2.4, Joseph (1976) is followed in a general discussion on stability bounds. Some more mathematical issues are placed in technical boxes and can be skipped on first reading.

2.1. Basic Equations

Standard notation as used in the field of geophysical fluid dynamics (such as in Pedlosky (1987)) is adopted. All dimensional dependent variables have a * subscript. This is useful to distinguish dimensionless and dimensional equations in later sections. The inner product is just indicated with a dot (.) and for the vector product, the $\wedge$ notation is used.

2.1.1. Coordinate free

Consider a flow of water within a bounded region $\mathcal{V}$ on Earth's sphere, an example shown in Fig. 2.1. The region rotates with the movement of the Earth, having rotation vector $\mathbf{\Omega}$ and angular frequency $\Omega = \mid \mathbf{\Omega} \mid$. The equations of motion described from a reference frame moving along with the earth are (Pedlosky, 1987)

$$\rho_* \left[\frac{D\mathbf{v}_*}{dt_*} + 2\mathbf{\Omega} \wedge \mathbf{v}_* \right] = -\nabla p_* + \rho_* \nabla \Phi + \rho_* \mathcal{F}_{I*} \quad (2.1a)$$

$$\frac{D\rho_*}{dt_*} + \rho_* \nabla . \mathbf{v}_* = 0 \quad (2.1b)$$

Here, $D/dt_* = \partial / \partial t_* + \mathbf{v}_* . \nabla$ is the material derivative. The vector $\mathbf{v}_*$ is the velocity field of the flow, p_* is the pressure field, and ρ_* is the density of the water. The quantity Φ is the geopotential, where the dominant term is given by the gravitational acceleration. In spherical coordinates $-\nabla \Phi = g \mathbf{i}_r$, with g the acceleration due to gravity and $\mathbf{i}_r$ the unit vector in radial-direction. The vector $\mathcal{F}_{I*}$ $[ms^{-2}]$ represents the accelerations due to random motions (mixing) and its form will be discussed in section 2.1.3.

Although this set-up is general, an approximation which is made in nearly all modelling studies is the Boussinesq approximation. In this approximation, only the effect of density differences is considered in the volume (e.g., gravity) force,