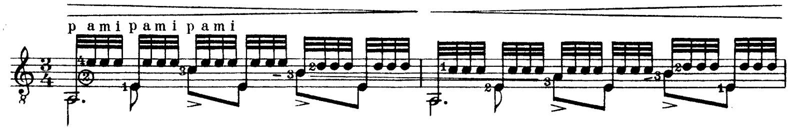
Chapter 4

NUMERICAL TECHNIQUES



For progress, certain skills are necessary.
Recuerdos d'Alhambra, F. Tarrega

If one wants to use the systematic methodology of chapter 3 on meaningful ocean and climate models, one must be able to determine at least the codimension-1 bifurcation points. As will become clear in this chapter, the detail of the dynamical behavior which can be analyzed depends on the dimension, say N, of the dynamical system. For systems of ordinary differential equations of small dimension ($N < 10$), the origin of very complex spatial and temporal dynamics can be investigated. For example, codimension-2 bifurcations can be determined numerically by software packages such as CONTENT (Kuznetsov, 1995) (available from http://ftp.cwi.nl/CONTENT/), DSTOOL (Guckenheimer and Kim, 1991) (available from http://www.cam.cornell.edu/guckenheimer/dstool.html) and MATCONT (available from http://allserv.rug.ac.be/∼ajdhooge/research.html)

For somewhat larger dimensional models, with dimensions up to $N = 100$, also software is available to perform analysis of the bifurcation behavior of the model, but the detail of analysis becomes already less. One highly recommended code is AUTO (Doedel, 1980) which is available from http://indy.cs.concordia.ca/auto/. In the very clear manual, the many capabilities of this program are described. For news on these packages, see for example http://www.amsta.leeds.ac.uk/Applied/news.dir/bifurcation.html.

For systems of partial differential equations, such as arising from ocean models (typically $N = 10^5$), two public domain packages is available. First package is the code PDECONT, described in Lust *et al.* (1998) and Lust and Roose (2000), which can be obtained through http://www.cs.kuleuven.ac.be/∼kurt/r_PDEcont.html. The package LOCA is developed by Andy Salinger at Sandia National Laboratories and is available through http://www.cs.sandia.gov/loca/.

In this chapter, numerical techniques to apply bifurcation analysis to large-dimensional dynamical systems will be described. Doedel and Tuckermann (2000) provide an overview of the many techniques around. The aim here is to sketch the methods available in the code (BOOM) which has been developed in Utrecht over the years and will be further developed in Fort Collins (see http://fractal.atmos.colostate.edu). It simultaneously provides a background on the computational approaches used in subsequent chapters and hopefully a good entrance to the literature for readers interested to pursue this subject further.

The starting point is a given set of partial differential equations which can be written in operator form as

$$\mathcal{M}\frac{\partial \mathbf{u}}{\partial t} + \mathcal{L}\mathbf{u} + \mathcal{N}(\mathbf{u}) = \mathbf{F} \tag{4.1}$$

where $\mathcal{L}$, $\mathcal{M}$ are linear operators, $\mathcal{N}$ is a nonlinear operator, $\mathbf{u}$ is the vector of dependent quantities and $\mathbf{F}$ contains the forcing of the system. To get a well-posed problem, appropriate boundary conditions have to be added to this set of equations. A typical problem will be given in section 4.1 and this problem also serves as a testcase for illustrating the methods.