Chapter 3

THE RASCH MODEL EXPLAINED

David Andrich
Murdoch University

Abstract: This Chapter explains the Rasch model for ordered response categories by demonstrating the latent response structure and process compatible with the model. This is necessary because there is some confusion in the interpretation of the parameters and the possible response process characterised by the model. The confusion arises from two main sources. First, the model has the initially counterintuitive properties that (i) the values of the estimates of the thresholds defining the boundaries between the categories on the latent continuum can be reversed relative to their natural order, and (ii) that adjacent categories cannot be combined in the sense that their probabilities can be summed to form a new category. Second, two identical models at the level of a single person responding to a single item, the so called rating and partial credit models, have been portrayed as being different in the response structure and response process compatible with the model. This Chapter studies the structure and process compatible with the Rasch model, in which subtle and unusual distinctions need to be made between the values and structure of response probabilities and between compatible and determined relationships. The Chapter demonstrates that the response process compatible with the model is one of classification in which a response in any category implies a latent response at every threshold. The Chapter concludes with an example of a response process that is compatible with the model and one that is incompatible.

Key words: rating credit models, partial credit models, Guttman structure, combing categories

1. INTRODUCTION

This Chapter explains the Rasch model for ordered response categories in standard formats by demonstrating the latent response structure and process compatible with the model. Standard formats involve one response in one of
the categories deemed a-priori to reflect increasing levels of the latent trait common in quantifying attitude, performance, and status in the social sciences. Table 3-1 shows such formats for four ordered categories. Later in the paper, a response format not compatible with the model is also shown.

<table>
<thead>
<tr>
<th>Table 3-1. Standard response formats for the Rasch model</th>
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<tbody>
<tr>
<td>Fail &lt; Pass &lt; Credit &lt; Distinction</td>
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<tr>
<td>Never &lt; Sometimes &lt; Often &lt; Always</td>
</tr>
<tr>
<td>Strongly Disagree &lt; Disagree &lt; Agree &lt; Strongly Agree</td>
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The response structure and process characterized by the Rasch model concerns the response of one person to one item in one of the categories. These categories are deemed a-priori to reflect increasing levels of the latent trait. This is evident in each of the formats in Table 3-1. The first example in Table 3-1, the assessment of performance in the ordered categories of Fail < Pass < Credit < Distinction, is used for further illustrations in the Chapter.

1.1 The original form of the model

The model of concern was derived originally (Andrich, 1978a) by expressing the general model studied by Rasch (1961) and Andersen (1977),

\[ \Pr \{ X = x \} = \frac{1}{\gamma} \exp(\kappa_x + \varphi_x(\beta - \delta)) \]  

in terms of thresholds resolved from the general scoring functions \( \varphi_x \) and category coefficients \( \kappa_x \) as

\[ \Pr \{ X = x, x > 0 \} = \frac{1}{\gamma} \exp(- \sum_{k=1}^{x} t_k + x(\beta - \delta)); \Pr \{ X = 0 \} = \frac{1}{\gamma} \]  

where (i) \( X = x \) is an integer random variable characterizing \( m + 1 \) successive categories which imply increasing values on the latent trait, (ii) \( \beta \) and \( \delta \) are respectively locations on the same latent continuum of a person and an item, (iii) \( t_k, k = 1,2,3,...m \) are \( m \) thresholds which divide the continuum into to \( m+1 \) ordered categories and which, without loss of

\[ \varphi_x = x; \kappa_x = - \sum_{k=1}^{x} t_k; \kappa_0 = 0 \]