CHAPTER 16

ELASTIC INSTABILITY (BUCKLING) OF SANDWICH PLATES

16.1 General Considerations

As stated previously, structures usually fail in one of four ways:

- overstressing (strength critical structure)
- over deflection (stiffness critical structure)
- resonant vibration
- buckling.

For many cases, because sandwich structures (compared to monocoque structures) minimize stresses, are extremely stiff, and have high fundamental natural vibration frequencies, care must be taken to insure that unanticipated buckling does not undermine a structural design.

In monocoque structures for given plate dimensions, material, boundary conditions, and a given load type (in-plane compression, in-plane shear), only one buckling load will result in actual buckling. This is the lowest eigenvalue of a countable infinity of such eigenvalues. All other eigenvalues exist mathematically, but only the lowest value has physical significance. This differs from natural frequencies in which several eigenvalues can be very important.

For the simplest cases, for columns and isotropic plates, an introduction was given in Chapter 6. While philosophically the simple examples cover the topic of buckling; more complex structures can have several types of buckling instabilities, any one of which can destroy the structure.

Historically, there have been four major textbooks dealing primarily with elastic stability or buckling. These are authored by Timoshenko and Gere [6.1], Bleich [6.2], Brush and Almroth [12.1] and Simitses [12.2]. A new text by Jones [6.4] will supplement these four. Although these texts deal primarily with structures other than sandwich, the solutions can be applied by using the appropriate flexural stiffnesses.

16.2 The Overall Buckling of an Orthotropic Sandwich Plate Subjected to In-Plane Loads - Classical Theory

From previous developments, it was seen that for a plate there are five equations associated with the in-plane stress resultants $N_x$, $N_y$, and $N_{xy}$ and the in-plane displacements they cause, namely $u_0$ and $v_0$. For the isotropic rectangular plate, see (2.50)-(2.54), and the isotropic circular plate, see (5.15), (5.16) and (5.25) through (5.27). For a composite material plate, the in-plane equilibrium equations are given by Equations
(11.6) and (11.7). From Equation (10.66), for the case of mid-plane symmetry \((B_y = 0)\) and no thermal or moisture considerations it is seen that the in-plane constitutive equations are:

\[
N_x = A_{11} \varepsilon_x^0 + A_{12} \varepsilon_y^0 + 2A_{16} \gamma_{xy}^0
\]

\[(16.1)\]

\[
N_y = A_{12} \varepsilon_x^0 + A_{22} \varepsilon_y^0 + 2A_{26} \gamma_{xy}^0
\]

\[(16.2)\]

\[
N_{xy} = A_{16} \gamma_{xy}^0 + A_{26} \gamma_{xy}^0 + 2A_{56} \gamma_{xy}^0
\]

\[(16.3)\]

For this case, all of the equations of Section 12.2 apply, simply by using the sandwich stiffness properties for the \(D_{ij}\) that are found in Section 15.1.

Likewise, for the mid-plane symmetric panel, the six governing equations involving \(M_x, M_y, M_{xy}, Q_x, Q_y\) and \(w\), are given by Equations (11.9), (11.11), (11.12), (11.17), (11.18), and (11.19), the latter three neglecting the \(D_{16}\) and \(D_{26}\) terms. One can see there is no coupling between in-plane and lateral action for the plate with mid-plane symmetry. Yet it is well known and often observed that in-plane loads do cause lateral deflections through buckling, which is usually disastrous.

The answer to the paradox is that in the above discussion only linear elasticity theory is considered, while the physical event of buckling is a non-linear problem. For brevity, the development of the non-linear theory will not be included herein because it is included in so many other texts, such as those cited in Section 16.1.

The results of including the terms to predict the advent or inception of buckling for the beam and plate are, modifying Equation (11.26), shown previously as (12.4),

\[
D_1 \frac{\partial^4 w}{\partial x^4} + 2D_3 \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_2 \frac{\partial^4 w}{\partial y^4} = p(x, y) + N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2}
\]

\[(16.4)\]

where clearly there is a coupling between the in-plane loads and the lateral deflection. For overall buckling of a sandwich panel the \(D_1, D_2\) and \(D_3\) flexural stiffnesses are given by (15.1) through (15.8).

It should be noted that the buckling loads, like the natural frequencies, are independent of the lateral loads, which will be disregarded in what follows. However, in actual structural analysis, the effect of lateral loads, in combination with the in-plane loads could cause overstressing and failure before the in-plane buckling load is reached. However, the buckling load is still independent of the type or magnitude of the lateral load, as are the natural frequencies. Incidentally, common sense dictates that if one is designing a structure to withstand compressive loads, with the possibility of buckling being the failure mode, one had better design the structure to be mid-plane symmetric, so