CHALLENGES IN USING SPLITTING TECHNIQUES FOR LARGE-SCALE ENVIRONMENTAL MODELING

Ivan Dimov\textsuperscript{(1)}, Tzvetan Ostromsky\textsuperscript{(1)}, and Zahari Zlatev\textsuperscript{(2)}

\textsuperscript{(1)}Institute for Parallel Processing, Bulgarian Academy of Sciences, Acad. G Bonchev str., bl. 25-A, 1113 Sofia, Bulgaria; \textsuperscript{(2)}National Institute for Environmental Research, Frederiksborgvej 399, Roskilde, Denmark

Abstract: Splitting techniques are, to our knowledge, used in all operationally run large scale air pollution models with many scenarios. The modellers believe that really huge computational tasks can be made tractable on the available computers by dividing them into a sequence of smaller and simple sub-tasks. Probably, the first simple splitting procedure for partial differential equations was proposed by Bagrinovski and Godunov in 1957. Since then many different splitting schemes were proposed and studied. A significant progress in splitting analysis was done during the last 3-4 years when the Laplace transformations technique was replaced by more powerful techniques. Several splitting schemes for large scale air pollution models are analysed and tested in a set of numerical experiments with respect to the splitting error. The computational properties of splitting schemes are analysed from algorithmic point of view. Parallel properties of these splitting schemes are also discussed.

Key words: Air quality, weather, numerical forecast, atmospheric chemistry, atmospheric transport.

1. INTRODUCTION

The application of splitting procedures in the treatment of large scientific and engineering problems is an excellent tool (and, very often, the only tool) by which huge computational tasks can be made tractable on the available computers. This is achieved by dividing the original problem into a sequence of simpler tasks.
A simple splitting procedure for partial differential equations (PDE) was proposed, as an example, by Bagrinovskii and Godunov (1957). This was probably the first attempt to introduce the splitting technique. Different splitting procedures have been developed and/or studied in many scientific papers, for example: Dyakonov (1962), Yanenko (1962), Marchuk (1968), Strang (1968), Penenko and Obraztsov (1976), Tikhonov and Samarski (1977), Marchuk (1980, 1986), Lanser and Verwer (1999), Faragó and Havasi (2002), Csomós et al. (2003). A detailed theoretical study and analysis of some splitting procedures can be found in Marchuk (1980), Marchuk (1988), Faragó and Havasi (2002), Faragó et al. (2002). Some convergence results were recently presented in Faragó and Havasi (2003).

Splitting techniques are successfully used in large-scale air pollution modeling (see McRae (1982), Marchuk (1986), Zlatev (1995), Dimov et al. (1999, 2001), Hunsdorfer and Verwer (2003)). Results related to the use of some splitting procedures in a large air pollution model (DEM in particular) as well as the advantages and disadvantages of these procedures are discussed in this paper.

2. GENERAL DESCRIPTION OF SPLITTING PROCEDURES

Consider the differential equation

\[ \frac{\partial c(x,t)}{\partial t} = A(c(x,t)) \]

(1)

where \( t \in [0, T] \), \( x \in \mathbb{D} \) and \( A \) is a differential operator.

We shall assume that operator \( A(c) \) can be represented as a sum of \( m \) operators, which in some sense simpler than the original operator \( A(c) \):

\[ A(c) = \sum_{i=1}^{m} A^{(i)}(c) \]

It should be noted here that the division of the operator \( A(c) \) to a sum of simpler operators is in general not unique.

This means that the original problem can be split into the following \( m \) subproblems:

\[ \frac{\partial c^{(i)}(x,t)}{\partial t} = A^{(i)}(c^{(i)}(x,t)), \quad i=1,2,\ldots,m. \]

(2)

Moreover, the time-interval \([0,T]\) is divided to \( N=T/\tau \) “small” sub-intervals, where \( \tau \) is called the splitting time-step. The particular splitting procedure depends on the way in which the operator \( A(c) \) is represented as a