BINARY DECISION DIAGRAMS AS A NEW PARADIGM FOR MORPHOLOGICAL MACHINES

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Abstract Mathematical Morphology (MM) is a general framework for studying mappings between complete lattices. In particular, mappings between binary images that are translation invariant and locally defined within a window are of special interest in MM. They are called W-operators. A key aspect of MM is the representation of W-operators in terms of dilations, erosions, intersection, union, complementation and composition. When W-operators are expressed in this form, they are called morphological operators. An implementation of this decomposition structure is called morphological machine (MMach). A remarkable property of this decomposition structure is that it can be represented efficiently by graphs called Binary Decision Diagrams (BDDs). In this paper, we propose a new architecture for MMachs that is based on BDDs. We also show that reduced and ordered BDDs (ROBDDs) are non-ambiguous schemes for representing W-operators and we present a method to compute them. This procedure can be applied for the automatic proof of equivalence between morphological operators, since the W-operator they represent are equal if and only if they have the same ROBDD.

Keywords: Binary Decision Diagram, Morphological Machine, Morphological Language, Morphological Operator.

1. Introduction

Mathematical Morphology (MM) is a theory that studies images and signals based on transformations of their shapes. These transformations can be viewed as mappings between complete lattices [8, 13]. In particular, mappings between binary images are of special interest in MM and they are called *set operators*. A central paradigm in MM is the representation of set operators in

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terms of dilations, erosions, union, intersection, complementation and composition. This decomposition structure can be described by a formal language called *morphological language* [2, 3]. Sentences of the morphological language are called *morphological operators* or *morphological expressions*. An implementation of the morphological language is called *morphological machine* (MMach). The first known MMach was the Texture Analyzer, created in the late sixties in Fontainebleau by Serra and Klein. Nowadays, a large number of these machines are available.

The motivation of this work comes from the search for non-ambiguous and compact representation for a large class of operators. It is also desired that this representation leads to efficient algorithms for morphological image processing and that it satisfactorily solves the issue of determining whether two representations are equivalent. In this context, we propose a new architecture for MMachs implemented as software for sequential machines. This new architecture is based on the representation of Boolean functions by Binary Decision Diagrams (BDDs) and was first used in MM in a special algorithm to compute the thinning operator [12].

An important class of set operators is that of W-operators, i.e., set operators that share the properties of translation invariance and local definition within a window. W-operators are extensively used in morphological image processing and this family of operators is the focus of this paper. This paper extends the use of BDD as a representation scheme for the whole class of W-operators.

The class of BDDs studied here (reduced and ordered BDDs) provides a trivial algorithm for determining whether morphological operators are equivalent. Algorithms to convert sentences of the morphological language to this new form of representation are also presented in this work.

2. Binary Mathematical Morphology

In this section, we recall some basic concepts of binary MM. Let E be a nonempty set and let $\mathcal{P}(E)$ denote the power set of E. Let $\subseteq$ denote the usual set inclusion relation. The pair $(\mathcal{P}(E), \subseteq)$ is a *complete Boolean lattice* [4]. A *set operator* is any mapping defined from $\mathcal{P}(E)$ into itself. The set $\boldsymbol{\Psi}$ of all set operators inherits the complete lattice structure of $(\mathcal{P}(E), \subseteq)$ by setting $\psi_1 \leq \psi_2 \Leftrightarrow \psi_1(X) \subseteq \psi_2(X), \forall \psi_1, \psi_2 \in \boldsymbol{\Psi}, \forall X \in \mathcal{P}(E)$. Let $X, Y \in \mathcal{P}(E)$. The operations $X \cup Y$, $X \cap Y$ and $X \backslash Y$, X^c are the usual set operations of union, intersection, difference and complementation, respectively.

Let $(E, +)$ be an *Abelian group* with zero element $o \in E$, called *origin*. Let $h \in E$ and $X, B \subseteq E$. The set X_h, defined by $X_h = \{x + h : x \in X\}$, is the *translation* of X by h. The set $X^t = \{-x : x \in X\}$ is the *transpose* of X. The set operations $X \oplus B = \cup_{b \in B} X_b$ and $X \ominus B = \cap_{b \in B} X_{-b}$ are the *Minkowski addition* and *Minkowski subtraction*, respectively.