VECTOR-ATTRIBUTE FILTERS

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Abstract A variant of morphological attribute filters is developed, in which the attribute on which filtering is based, is no longer a scalar, as is usual, but a vector. This leads to new granulometries and associated pattern spectra. When the vector-attribute used is a shape descriptor, the resulting granulometries filter an image based on a shape or shape family instead of one or more scalar values.

Keywords: Mathematical morphology, connected filters, multi-scale analysis, granulometries, pattern spectra, vector-attributes, shape filtering

Introduction

Attribute filters [2, 12], which preserve or remove components in an image based on the corresponding attribute value, are a comparatively new addition to the image processing toolbox of mathematical morphology. Besides binary and gray-scale 2-D images [2, 12], these filters have also been extended to handle vector images, like color images [5, 7] and tensor-valued data [3], and 3-D images. So far the attributes used in all of these cases have been scalars. Although the set of scalar attributes used in multi-variate filters and granulometries [14] can also be considered as a single vector-attribute, these multi-variate operators can always be written as a series of uni-variate scalar operators, which is not the case for vector-attribute filters.

In this paper vector-attribute filters and granulometries will be introduced, whose attributes consists of vectors instead of scalar values, followed by a discussion on their use as filters and in granulometries where the parameter is a single shape image or a family of shape images instead of a threshold value.

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1. Theory

The theory of granulometries and attribute filters is presented only very briefly here. For more detail the reader is referred to [2, 9, 12, 16]. In the following discussion binary images X and Y are defined as subsets of the image domain $\mathbf{M} \subset \mathbb{R}^n$ (usually $n = 2$), and gray-scale images are mappings from $\mathbf{M}$ to $\mathbb{R}$.

Let us define a scaling X_λ of set X by a scalar factor $\lambda \in \mathbb{R}$ as

$$X_\lambda = \{x \in \mathbb{R}^n | \lambda^{-1} x \in X\}. \tag{1}$$

An operator ϕ is said to be *scale-invariant* if

$$\phi(X_\lambda) = (\phi(X))_\lambda \tag{2}$$

for all $\lambda > 0$. A scale-invariant operator is therefore sensitive to shape rather than to size. If an operator is scale, rotation and translation invariant, we call it a *shape operator*. A *shape filter* is simply an idempotent shape operator. In the digital case, pure scale invariance will be harder to achieve due to discretization artefacts, but a good approximation may be achieved.

Attribute openings and thinnings

Attribute filters, as introduced by Breen and Jones [2], use a criterion to remove or preserve connected components (or flat zones for the gray-scale case) based on their attributes. The concept of trivial thinnings Φ_T is used, which accepts or rejects connected sets based on a non-increasing criterion T. A criterion T is increasing if the fact that C satisfies T implies that D satisfies T for all $D \supset C$. The binary connected opening $\Gamma_x(X)$ of set X at point $x \in \mathbf{M}$ yields the connected component of X containing x if $x \in X$, and $\emptyset$ otherwise. Thus Γ_x extracts the connected component to which x belongs, discarding all others. The trivial thinning Φ_T of a connected set C with criterion T is just the set C *if* C satisfies T, and is empty otherwise. Furthermore, $\Phi_T(\emptyset) = \emptyset$.

DEFINITION 1 *The binary attribute thinning Φ^T of set X with criterion T is given by*

$$\Phi^T(X) = \bigcup_{x \in X} \Phi_T(\Gamma_x(X)) \tag{3}$$

It can be shown that this is a thinning because it is idempotent and anti-extensive [2]. The attribute thinning is equivalent to performing a trivial thinning on all connected components in the image, i.e., removing all connected components which do not meet the criterion. It is trivial to show that if criterion