LABELLED RECONSTRUCTION OF BINARY OBJECTS: A VECTOR PROPAGATION ALGORITHM

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Abstract The *quench function* of a binary image is the distance transform of the image sampled on its skeleton. In principle the original image can be *reconstructed* from the quench function by drawing a disk at each point on the skeleton with radius given by the corresponding quench function value. This reconstruction process is of more than theoretical interest. One possible use is in coding of binary images, but our interest is in an applied image analysis context where the skeleton has been (1) *reduced* by, for example, deletion of barbs or other segments, and/or (2) *labelled* so that segments, or indeed individual pixels, have identifying labels. A useful reconstruction, or partial reconstruction, in such a case would be a labelled image, with labels propagated from the skeleton in some intuitive fashion, and the support of this labelled output would be the theoretical union of disks.

An algorithm which directly draws disks would, in many situations, be very inefficient. Moreover the label value for each pixel in the reconstruction is highly ambiguous in most cases where disks are highly overlapping. We propose a vector propagation algorithm based on Ragnelmalm's Euclidean distance transform algorithm which is both efficient and provides a natural label value for each pixel in the reconstruction. The algorithm is based on near-exact Euclidean distances in the sense that the reconstruction from a single-pixel skeleton is, to a very good approximation, a Euclidean digital disk. The method is illustrated using a biological example of neurite masks originating from images of neurons in culture.

Keywords: debarbing, Euclidean distance transform, object reconstruction, quench function, skeleton, vector propagation

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1. Introduction

This paper deals with the process of *reconstruction* of a binary image using a sub-sample of the (Euclidean) *distance transform* of the image. Figure 1 shows part of a binary image representing neurites in an image of neurons in culture, while Figure 2 shows the Euclidean distance transform (EDT) of this image sampled on a *skeleton* of the image, with darker pixels corresponding to higher values of the distance transform. When, as here, a distance transform is sampled on a skeleton, the result is called a *quench function* [11, p. 159].

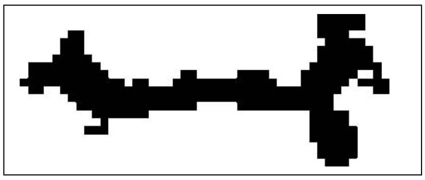

Figure 1. Neurite Mask.

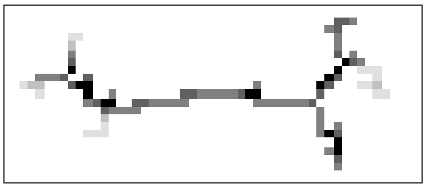

Figure 2. Quench Function.

The original mask, Figure 1, can be *reconstructed* from the quench function provided the skeleton is, in some sense, sufficiently complete. This can, in principle, be achieved by drawing a binary disk at each point on the skeleton with radius given by the quench function value at that point. While this process may be useful for coding and decoding of binary image data, it is of theoretical interest only in a typical context of analysis of images where the original mask is already known anyway. A more interesting case is shown in Figures 3 and 4. The reduced quench function in Figure 3 has been obtained from the full quench function in Figure 2 by removing some of the smaller "barbs". The topology has slightly changed for complicated reasons in this particular example; the details are not relevant to this discussion.

The binary image in Figure 4 is a *partial reconstruction* of the original image produced by the new algorithm described in this paper using the reduced quench function. It may be seen that most but not all of the original image has been reconstructed. This is a first application of this kind of algorithm, namely filtering of binary masks of linear features such as neurite networks. Such networks can be complex and noisy. The process illustrated here cleans and simplifies a network mask essentially by (1) computing the quench function of the mask, (2) topologically simplifying (e.g. debarbing) the quench function, and finally (3) reconstructing a reduced mask from the reduced quench function.

Figures 5 and 6 illustrate another important aspect of the new algorithm, namely *label propagation*. Figure 5 shows a labelled skeleton corresponding to the (reduced) quench function in Figure 3. That is, Figures 3 and 5 are two positive valued functions defined on the same set of skeleton points, Figure 3